Estimation and effects of the mass outflow
from shock compressed flow around compact objects

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Abstract. Outflows are common in many astrophysical systems which contain black holes and neutron stars. Difference between stellar outflows and outflows from these systems is that the outflows in these systems have to form out of the inflowing material only. The inflowing material can form a hot and dense cloud surrounding the compact object, either because of a centrifugal barrier, or a denser barrier due to pair plasma or pre-heating effects. This barrier behaves like a stellar surface as far as the mass loss is concerned. We estimate the outflow rate from the regions of shock compressed flow. The outflow rate is directly related to the compression ratio of the gas at the shocks. These estimated rates roughly match the rates in real observations as well as those obtained from numerical experiments. In special geometries, where the solid angle of the outflow is higher, the disk evacuation takes place creating quiescence states. Outflows are shown to be important in deciding the spectral states and Quasi Periodic Oscillations (QPO) of observed X-rays coming out of compact objects.

Key words: accretion, accretion disks – black hole physics – shock waves – stars: neutron – stars: individual: SS433 – stars: winds, outflows

1. Introduction

Cosmic radio jets are believed to originate from the centers of active galaxies which harbor black holes (e.g., Chakrabarti 1996, hereafter C96). Even in so-called 'micro-quasars', such as GRS 1915+105 which are believed to have stellar mass black holes (Mirabel & Rodriguez 1994), the outflows are common. The well collimated outflow in SS433 has been well known for almost two decades (Margon 1984). Similarly, systems with neutron stars also show outflows, as is believed to be the case in X-ray bursters (e.g. Titarchuk 1994).

There is a large number of articles in the literature which attempt to explain the origin of these outflows. These articles can be broadly divided into three sets. In one set, the jets are believed to come out due to hydrodynamic or magneto-hydrodynamic pressure effects and are treated separately from the disks (e.g., Fukue 1982; Chakrabarti 1986). In another set, efforts are made to correlate the disk structure with that of the outflow (e.g.,

Königl 1989; Chakrabarti & Bhaskaran 1992). In the third set, numerical simulations are carried out to actually see how matter is deflected from the equatorial plane towards the axis (e.g., Hawley 1984; Eggum et al. 1985; Molteni et al. 1994). Nevertheless, the definitive understanding of the formation of outflows is still lacking, and more importantly, it has always been difficult to estimate the outflow rate from first principles. In the first set, the outflow is not self-consistently derived from the inflow. In the second set, only self-similar steady solutions are found and in the third set, either a Keplerian disk or a constant angular momentum disk was studied, neither being the best possible assumption. On the other hand, the mass outflow rates of the normal stars are calculated very accurately from the stellar luminosity. The theory of radiatively driven winds seems to be very well understood (e.g. Castor et al. 1975). Given that the black holes and the neutron stars are much simpler celestial objects, and the flow around them is sufficiently hot to be generally ionized, it should have been simpler to compute the outflow rate from an inflow rate by using the methods employed in stellar physics.

Our approach to the mass outflow rate computation is somewhat different from that used in the literature so far. Though we consider simple-minded equations to make our points, such as those applicable to conical inflows and outflows, we add a fundamental ingredient to the system, whose importance is being revealed only very recently in the literature. This is the quasi-spherical centrifugally supported dense atmosphere with a typical size of a few tens of Schwarzschild radiusus around a black hole and a neutron star. Whether a shock actually forms or not, this dense region exists, as long as the angular momentum of the flow close to the compact object is roughly constant and is generally away from a Keplerian distribution as is the case in reality (C96). This centrifugally supported region (which basically forms the boundary layer of black holes and weakly magnetized neutron stars) successfully replaced the so called 'Compton cloud' (Chakrabarti & Titarchuk 1995 [hereafter CT95]; Chakrabarti et al. 1996) in explaining hard and soft states of black hole candidates, and the converging flow property of this region successfully produced the power-law spectral slope in the soft states of black hole candidates (CT95). The oscillation of this region successfully explains the general properties of the quasi-periodic oscillation (Molteni et al. 1996, C96) of X-rays

from black holes and neutron stars. It is therefore of interest to know if this region plays any major role in the formation of outflows.

Several authors have also mentioned denser regions forming due to different physical effects. Chang & Ostriker (1985) showed that pre-heating of the gas could produce standing shocks at a large distance. Kazanas & Ellison (1986) mentioned that pressure due to pair plasma could produce standing shocks at smaller distances around a black hole as well. Our computation is insensitive to the actual mechanism by which the boundary layer is produced. All we require is that the gas should be hot in the region where the compression takes place (i.e., the optical depth should be small). Thus, since Comptonization processes cool this region (CT95) for larger accretion rates ($\dot{M} \gtrsim 0.1\dot{M}_{Eddington}$) our process will produce outflows in hard states and low-luminosity objects consistent with current observations. Some workers talked about a so-called 'cauldron' model of compact objects where jets were assumed to emerge from a dense mixture of matter and radiation through a de-Laval nozzle as in the 'twin-exhaust' model (for a review of these models see Begelman et al. 1984). The difference between this model and the present one is that there a very high accretion rate was required ($\dot{M}_{in} \sim 1000\dot{M}_E$) while we consider thermally driven outflows from smaller accretion rates. Second, the size of the 'cauldron' was thousands of Schwarzschild radii (where gravity was so weak that the channel has to have the shape of a de-Laval nozzle), while we have a CENBOL of about $10R_g$ (where the gravity plays an active role in creating the transonic wind) in our mind. Third, in the present case, matter is assumed to pass through a sonic point using the pre-determined funnel where rotating pre-jet matter is accelerated (Chakrabarti 1984) and not through a 'bored nozzle' even though symbolically a quasi-spherical CENBOL is considered for mathematical convenience. Fourth, for the first time we compute the outflow rate completely analytically starting from the inflow rate alone. To our knowledge such a calculation has not been done in the literature at all.

Once the presence of our centrifugal-pressure -supported boundary layer (CENBOL) is accepted, the mechanism of the formation of the outflow becomes clearer. One basic criterion is that the outflowing winds should have a positive Bernoulli constant (C96) (although in the presence of radiative momentum deposition, a flow with negative initial energy could also escape as outflow, see Chattopadhyay & Chakrabarti 1999). Just as photons from the stellar surface deposit momentum on the outflowing wind and keep the flow roughly isothermal at least up to the sonic point, one may assume that the outflowing wind close to the black hole is kept isothermal due to deposition of momentum from hard photons. In the case of the sun, its luminosity is only $10^{-5} L_{Edd}$ and the typical mass outflow rate from the solar surface is $10^{-14}M_\odot$ year^{-1} (Priest 1982). Proportionately, for a star with an Eddington luminosity, the outflow rate would be $10^{-9}M_\odot$ year^{-1}. This is roughly half the Eddington rate for a stellar-mass star. Thus, if the flow is compressed and heated at the centrifugal barrier around a black hole, it would also radiate enough to keep the flow isothermal (at least up to

the sonic point) if the efficiency were exactly identical. Physically, both requirements may be equally difficult to meet, but in reality with photons shining on outflows near a black hole with almost 4π solid angle (from the funnel wall) it is easier to maintain the isothermality in the slowly moving (subsonic) region in the present context. Another reason is this: the process of momentum deposition on electrons is more efficient near a black hole. The electron density n_e falls off as $r^{-3/2}$ while the photon density n_γ falls off as r^{-2}. Thus the ratio $n_e/n_\gamma \propto r^{1/2}$ increases with the size of the region. Thus a compact object will have a lesser number of electrons per photon and the momentum transfer is more efficient. In a simpler-minded way, the physics is scale-invariant, though. In solar physics, it is customary to choose a momentum deposition term which keeps the flow isothermal to be of the form (Kopp & Holzer 1976; Chattopadhyay & Chakrabarti 1999),

$$F_r = \int_{R_s}^{r} D dr,$$

where D is the momentum deposition (localized around r_p) factor with a typical spatial dependence,

$$D = D_0 e^{-\alpha(r/r_p-1)^2}.$$

Here, D_0, α are constants and R_s is the location of the stellar surface. Since r and r_p appear in ratio, exactly the same physical consideration would be applicable to black hole physics, with the same result *provided* D_0 is scaled with luminosity. (But, as we showed above, D_0 increases for a compact object.) However, as CT95 showed, a high accretion rate ($\dot{M} \gtrsim 0.3\dot{M}_{Edd}$) will *reduce* the temperature of the CENBOL catastrophically, and therefore our assumption of isothermality of the outflow would break down at these high rates. It is to be noted that in the context of stellar physics, it is shown (Pauldrach et al. 1986) that the temperature stratification in the outflowing wind has little effect on the mass loss rate.

Having thus been convinced that isothermality of the outflow, at least up to the sonic point, is easier to maintain near a black hole, we present in this paper a simple derivation of the ratio of the mass outflow rate and mass inflow rate assuming the flow is externally collimated. We find that the ratio is a function of the compression ratio of the gas at the boundary of the hot, dense, centrifugally supported region. We estimate that the outflow rate should generally be less than a few percent if the outflow is well collimated. In Sect. 3, we find some interesting effects when the region out to the sonic point of the outflow cools down periodically and causes periodic change of spectral states. Finally, in Sect. 4, we draw our conclusions.

2. Derivation of the outflow rate

Fig. 1a shows the schematic nature of the inflow and outflow that is understood to be taking place around a black hole. Rotational motion brakes the flow and forms a dense boundary layer (CENBOL) around a black hole. Matter compressed and heated (over and above the heating due to geometric compression) at the CENBOL comes out in between the centrifugal
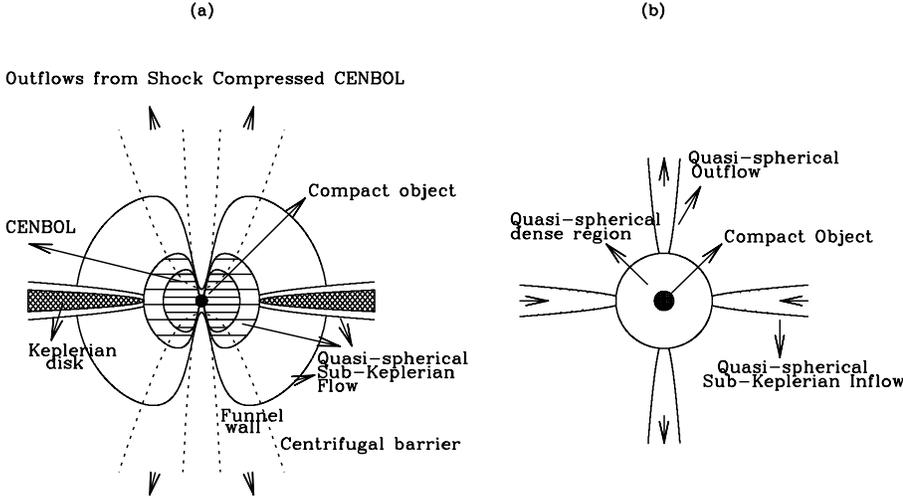
(a)   (b)

Outflows from Shock Compressed CENBOL

Fig. 1a and b. Schematic diagram of inflow and outflow around a compact object. The hot, dense region around the object either due to a centrifugal barrier or due to a plasma pressure effect or pre-heating, acts like a 'stellar surface' from which the outflowing wind is developed. In **a**, the actual flow behavior is shown. In **b**, a simplified model is depicted where the toroidal CENBOL is replaced by a quasi-spherical CENBOL.

barrier and the funnel wall (Molteni et al. 1994, C96). For the present paper, in Fig. 1b, we choose a simplified description of this system, where the toroidal CENBOL is replaced by a quasi-spherical one. Matter is assumed to fall in a conical shape. The sub-Keplerian, hot and dense, quasi-spherical region may also form due to pair-plasma pressure or pre-heating effects, but the details are not essential. The outflowing wind is assumed to be also conical in shape for simplicity and is flowing out along the axis. It is assumed that the wind is collimated by an external pressure. Both the inflow and the outflow are assumed to be thin enough so that the velocity and density variations across the flow could be ignored.

The accretion rate of the incoming accretion flow is given by,

$$\dot{M}_{in} = \Theta_{in}\rho\vartheta r^2. \tag{1}$$

Here, Θ_{in} is the solid angle subtended by the inflow, ρ and ϑ are the density and velocity respectively, and r is the radial distance. For simplicity, we assume geometric units ($G = 1 = M_{BH} = c$; G is the gravitational constant, M_{BH} is the mass of the central black hole, and c is the velocity of light) to measure all the quantities. In these units, for a freely falling gas,

$$\vartheta(r) = \left[\frac{1-\Gamma}{r}\right]^{1/2} \tag{2}$$

and

$$\rho(r) = \frac{\dot{M}_{in}}{\Theta_{in}}(1-\Gamma)^{-1/2}r^{-3/2} \tag{3}$$

Here, Γ/r^2 (with Γ assumed to be a constant) is the outward force due to radiation.

We assume that the boundary of the denser cloud (say, the shock) is at $r = r_s$ (typically a few to few tens of Schwarzschild radii, see Chakrabarti 1990 [hereafter C90], C96) where the inflow gas is compressed. The compression could be abrupt due to a standing shock or gradual as in a shock-free flow with angular momentum. These details are irrelevant. At this barrier, then

$$\rho_+(r_s) = R\rho_-(r_s) \tag{4a}$$

and

$$\vartheta_+(r_s) = R^{-1}\vartheta_-(r_s) \tag{4b}$$

where R is the compression ratio. The exact value of the compression ratio is a function of the flow parameters, such as the specific energy and the angular momentum (e.g. C90). Here, the subscripts $-$ and $+$ denote the pre-shock and post-shock quantities respectively. At the shock surface, the total pressure (thermal pressure plus ram pressure) is balanced.

$$P_-(r_s) + \rho_-(r_s)\vartheta_-^2(r_s) = P_+(r_s) + \rho_+(r_s)\vartheta_+^2(r_s). \tag{5}$$

Assuming that the thermal pressure of the pre-shock incoming flow is negligible compared to the ram pressure, using Eqs. (4a,b) we find,

$$P_+(r_s) = \frac{R-1}{R}\rho_-(r_s)\vartheta_-^2(r_s). \tag{6}$$

The isothermal sound speed in the post-shock region is then,

$$C_s^2 = \frac{P_+}{\rho_+} = \frac{(R-1)(1-\Gamma)}{R^2}\frac{1}{r_s} = \frac{(1-\Gamma)}{f_0 r_s} \tag{7}$$

where $f_0 = R^2/(R-1)$. An outflow which is generated from this dense region with very low flow velocity along the axis is necessarily subsonic in this region, however, at a large distance, the outflow velocity is expected to be much higher compared to the sound speed, and therefore the flow must be supersonic. In the subsonic region of the outflow, the pressure and density are expected to be almost constant and thus it is customary to assume isothermality conditions up to the sonic point. As argued in the introduction, in the case of black hole accretion also, such an assumption may be justified. With the isothermality assumption or a given temperature distribution ($T \propto r^{-\beta}$ with β a constant) the result is derivable in analytical form. The sonic point conditions are obtained from the radial momentum equation,

$$\vartheta\frac{d\vartheta}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{1-\Gamma}{r^2} = 0 \tag{8}$$

and the continuity equation

$$\frac{1}{r^2}\frac{d(\rho\vartheta r^2)}{dr} = 0 \tag{9}$$

in the usual way, i.e., by eliminating $d\rho/dr$,

$$\frac{d\vartheta}{dr} = \frac{N}{D} \tag{10}$$

where

$$N = \frac{2C_s^2}{r} - \frac{1-\Gamma}{r^2}$$

and

$$D = \vartheta - \frac{C_s^2}{\vartheta}$$

and putting $N = 0$ and $D = 0$. These conditions yield, at the sonic point $r = r_c$, for an isothermal flow,

$$\vartheta(r_c) = C_s \tag{11a}$$

and

$$r_c = \frac{1-\Gamma}{2C_s^2} = \frac{f_0 r_s}{2} \tag{11b}$$

where we have utilized Eq. (7) to substitute for C_s.

Since the sonic point of a hotter outflow is located closer to the black hole, clearly, the condition of isothermality is best maintained if the temperature is high enough. However if the temperature is too high, so that $r_c < r_s$, then the flow has to bore a hole through the cloud just as in the 'cauldron' model, although it is a different situation — here the temperature is high, while in the 'cauldron' model the temperature was low. In reality, a pre-defined funnel caused by a centrifugal barrier does not require the flow to bore any nozzle at all, but our simple quasi-spherical calculation fails to describe this case properly. This is done in detail in Das & Chakrabarti (1999).

The constancy of the integral of the radial momentum equation (Eq. (8)) in an isothermal flow gives:

$$C_s^2 ln\, \rho_+ - \frac{1-\Gamma}{r_s} = \frac{1}{2}C_s^2 + C_s^2 ln\, \rho_c - \frac{1-\Gamma}{r_c} \tag{12}$$

where we have ignored the initial value of the outflowing radial velocity $\vartheta(r_s)$ at the dense region boundary, and used Eq. (11a). We have also put $\rho(r_c) = \rho_c$ and $\rho(r_s) = \rho_+$. After simplification, we obtain,

$$\rho_c = \rho_+ exp(-f) \tag{13}$$

where,

$$f = f_0 - \frac{3}{2}.$$

Thus, the outflow rate is given by,

$$\dot{M}_{out} = \Theta_{out}\rho_c\vartheta_c r_c^2 \tag{14}$$

where Θ_{out} is the solid angle subtended by the outflowing cone. Upon substitution, one obtains,

$$\frac{\dot{M}_{out}}{\dot{M}_{in}} = R_{\dot{m}} = \frac{\Theta_{out}}{\Theta_{in}}\frac{R}{4}f_0^{3/2}exp\,(-f) \tag{15}$$
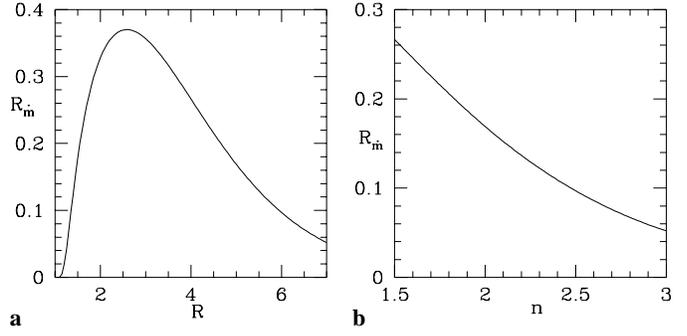

Fig. 2a and b. Ratio $\dot{R}_{\dot{m}}$ of the outflow rate and the inflow rate as a function of the compression ratio of the gas at the dense region boundary **a**. In **b**, variation of the ratio with the polytropic constant n in the strong shock limit is shown. Solid angles subtended by the inflow and the outflow are assumed to be comparable for simplicity.

which explicitly depends only on the compression ratio:

$$\frac{\dot{M}_{out}}{\dot{M}_{in}} = R_{\dot{m}} = \frac{\Theta_{out}}{\Theta_{in}}\frac{R}{4}\left[\frac{R^2}{R-1}\right]^{3/2}exp\left(\frac{3}{2} - \frac{R^2}{R-1}\right) \tag{16}$$

apart from the geometric factors. Notice that this simple result does not depend on the location of the sonic point or the the size of the shock or the outward radiation force constant Γ. This is because the Newtonian potential was used throughout and the radiation force was also assumed to be very simple-minded (Γ/r^2). Also, effects of centrifugal force were ignored. Similarly, the ratio is independent of the mass accretion rate which should be valid only for low luminosity objects. For high luminosity flows, Comptonization would cool the dense region completely (CT95) and the mass loss will be negligible. Pair-plasma-supported quasi-spherical shocks form for low luminosity as well (Kazanas & Ellison 1986). In reality there would be a dependence on these quantities when full general relativistic considerations of the rotating flows are made.

Figs. 2a and b contain the basic results. Fig. 2a shows the ratio $R_{\dot{m}}$ as a function of the compression ratio R (plotted from 1 to 7). Fig. 2b shows the same quantity as a function of the polytropic constant $n = (\gamma - 1)^{-1}$ (drawn from $n = 3/2$ to 3), γ being the adiabatic index. Fig. 2a is drawn for any generic compression ratio and Fig. 2b is drawn assuming the strong shock limit only: $R = (\gamma + 1)/(\gamma - 1) = 2n + 1$. In both the curves, $\Theta_{out} \sim \Theta_{in}$ has been assumed for simplicity. Note that if the compression (over and above the geometric compression) does not take place (namely, if the denser region does not form), then there is no outflow in this model. Indeed for, $R = 1$, the ratio $R_{\dot{m}}$ is zero as expected. Thus the driving force of the outflow is primarily coming from the hot and compressed region.

In a relativistic inflow or for a radiation dominated inflow, $n = 3$ and $\gamma = 4/3$. In the strong shock limit, the compression ratio is $R = 7$ and the ratio of inflow and outflow rates becomes

$$R_{\dot{m}} = 0.052\,\frac{\Theta_{out}}{\Theta_{in}}. \tag{17a}$$

For the inflow of a mono-atomic ionized gas $n = 3/2$ and $\gamma = 5/3$. The compression ratio is $R = 4$, and the ratio in this

case becomes

$$R_{\dot{m}} = 0.266 \, \frac{\Theta_{out}}{\Theta_{in}}. \tag{17b}$$

Since f_0 is smaller for a $\gamma = 5/3$ case, the density at the sonic point in the outflow is much higher (due to the exponential dependence of density on f_0, see Eq. (7)) which causes the higher outflow rate, even when the actual jump in density in the post-shock region, the location of the sonic point and the velocity of the flow at the sonic point are much lower. It is to be noted that generally for $\gamma > 1.5$ shocks are not expected (C90), but the centrifugal-barrier-supported dense region would still exist. As is clear, the entire behavior of the outflow depends only on the compression ratio, R and the collimating property of the outflow Θ_{out}/Θ_{in}.

Outflows are usually concentrated near the axis, while the inflow is near the equatorial plane. Assuming a half angle of 10° in each case, we obtain

$$\Theta_{in} = \frac{2\pi^2}{9}; \qquad \Theta_{out} = \frac{\pi^3}{162}$$

and

$$\frac{\Theta_{out}}{\Theta_{in}} = \frac{\pi}{36}. \tag{18}$$

The ratios of the rates for $\gamma = 4/3$ and $\gamma = 5/3$ are then

$$R_{\dot{m}} = 0.0045 \tag{19a}$$

and

$$R_{\dot{m}} = 0.023 \tag{19b}$$

respectively. Thus, in quasi-spherical systems, in the case of strong shock limit, the outflow rate is at most a few percent of the inflow. If this assumption is dropped, then for flow with a weaker shock the rate could be much higher (see Fig. 2a).

The angle Θ_{out} must be related to the collimation property of the ambient medium, as well as the strength of the angular momentum barrier, although it is doubtful if matter achieves the observed collimation right close to the black hole. The stronger the barrier is, the higher is Θ_{out} (Molteni et al. 1994) and therefore the higher the loss rate. If Θ_{out} is sufficiently high and Θ_{in} is low ($\Theta_{out} + \Theta_{in} = 4\pi$) the outflow may cause a *complete* evacuation of the disk causing a quiescence state of the black hole candidates. The peak area in Fig. 2a would have interesting effects on the relation between spectral states and quasi-periodic oscillations of spectra of black holes as will be discussed in Sect. 4.

It is to be noted that the above expression for the outflow rate is strictly valid if the flow could be kept isothermal at least up to the sonic point. If this assumption is dropped the expression for the outflow rate becomes dependent on several parameters. As an example, we consider a polytropic outflow of the same index γ but of a different entropy function K (we assume the equation of state to be $P = K\rho^\gamma$, with $\gamma \neq 1$) the expression (11b) would be replaced by

$$r_c = \frac{f_0 r_s}{2\gamma} \tag{20}$$

and Eq. (12) would be replaced by

$$na_+^2 - \frac{1-\Gamma}{r_s} = \left(\frac{1}{2} + n\right) a_s^2 - \frac{1-\Gamma}{r_c} \tag{21}$$

where $n = 1/(\gamma - 1)$ is the polytropic constant of the flow and $a_+ = (\gamma P_+/\rho_+)^{1/2}$ and $a_c = (\gamma P_c/\rho_c)^{1/2}$ are the adiabatic sound speeds at the starting point and the sonic point of the outflow. It is easily shown that a power law temperature falloff of the outflow ($T \propto r^{-\beta}$) would yield

$$R_{\dot{m}} = \frac{\Theta_{out}}{\Theta_{in}} \left(\frac{K_i}{K_o}\right)^n \left(\frac{f_0}{2\gamma}\right)^{\frac{3}{2} - \beta}, \tag{22}$$

where K_i and K_o are the entropy functions of the inflow and the outflow. This derivation is strictly valid for a non-isothermal flow. Since $K_i < K_o$, $n > 3/2$ and $f_0 > 2\gamma$, for $\Theta_{out} \sim \Theta_{in}$, $R_{\dot{m}} \ll 1$ is guaranteed provided $\beta > \frac{3}{2}$, i.e., if the temperature falls off sufficiently rapidly. For an isothermal flow $\beta = 0$ and the rate tends to be higher. Note that since $n \sim \infty$ in this case, any small jump in entropy due to compression will balance the effect of the $f_0^{-3/2}$ factor. Thus $R_{\dot{m}}$ remains smaller than unity. The first factor decreases with the entropy jump while the second factor will have a minimum at $R = 2$ when $\beta < 3/2$. Thus the solution is still expected to be similar to what is shown in Fig. 2a,b. Numerical results of the transonic flow using a non-isothermal equation of state are discussed elsewhere (Das & Chakrabarti 1999).

3. Effects of outflows from shocked compressed accretion

One can discuss the effects of outflows of a stellar black candidate such as GRS 1915+105. Particularly interesting is that it shows QPO of around 1–15Hz and sometimes flares at around 0.01Hz (Paul et al. 1998). An interesting property of our solution (Fig. 2a) is that the outflow rate is peaked at around $R = 2.5$ ($f_0 \sim 4$), when the shock is of 'average' strength. On either side, the outflow rate falls off very rapidly and this may have significant observational effects. As Eq. (16) is independent of shock location, shocks oscillating with time period similar to the infall time in $r < r_s$ region (causing a 1–10Hz QPO in the hard state (CT95, C96)) will continue to have outflows and gradually fill the relatively slowly moving (sonic) sphere of radius $r = r_c = f_0 r_s/2$ till $\langle\rho\rangle r_c \sigma_T \sim 1$ when this region would be cooled down catastrophically by inverse Comptonization. (In general, spectra may soften in the presence of outflows even in the hard state, see Chakrabarti, 1998) At this stage: (a) the $r < r_c$ region would be drained to the hole in $t_{fall} \sim r_c^{3/2} 2GM/c^2 = 0.1 \left(\frac{r_s}{50}\right)^{3/2} \left(\frac{M}{10M_\odot}\right) \left(\frac{f_0}{4}\right)$ seconds. This is typically what is observed for GRS1915+105. (Yadav et al. 1999). (b) The shock will disappear, and a smaller compression ratio ($R \rightarrow 1$) would stop the outflow (Fig. 2a). In other words, during burst/quiescence QPO phase, the outflow would be blobby. This is also reported (Mirabel & Rodriguez 1998). (c) The black hole will go to a soft state during a short period. If the angular momentum is high enough, so that the outflow rate is really high, this brief period of soft state may be prolonged to

a longer period of tens of seconds depending on how the centrifugal barrier is removed by viscosity generated during shock oscillations. The fact that shock oscillation causes the 1–15Hz QPO is clearly demonstrated by the fact there is more power at high energy (Fig. 3 of Paul et al. 1998) and most of the high energy X-ray radiation is emitted in the post-shock region.

Using our solution, it is easy to compute the interval between two bursts during which the object is in QPO phase with $\nu_{QPO} \sim 1$–10 Hz. The sonic sphere becomes ready for catastrophic cooling in

$$t_{b-b} = \frac{4/3\pi r_c^3 \langle\rho\rangle}{\dot{M}_{out}} \ s \sim \frac{160 r_s^2}{R_{\dot{m}}\dot{M}_{in}} \ s. \tag{23}$$

Here, $\langle\rho\rangle$ is the average density of the sonic sphere and $\langle\rho\rangle r_c \sigma_T \sim 1$, $\sigma_T = 0.4$ is the Thomson scattering cross-section. For an average shock R around 2.5, f_o stays close to 4, and with outflow and inflow of roughly equal angular dimension ($\theta \sim 45^o$), $\Theta_{out}/\Theta_{in} \sim 0.17$ and $R_{\dot{m}} \sim 0.06$ (using peak value in Fig. 3b). Putting typical values of a hard state $\dot{M} = 0.1\dot{M}_{Edd}$, and $r_s = 50$, $M = 10M_\odot$ in the above equation, we obtain

$$t_{b-b} = 42.5 \left(\frac{r_s}{50}\right)^2 \left(\frac{M}{10M_\odot}\right) \left(\frac{\dot{M}}{0.1\dot{M}_{Edd}}\right)^{-1} \ s. \tag{24}$$

Our choice of $r_s = 50$ is not arbitrary. If the viscosity is low, the angular momentum could be high enough to have a shock at a larger distance (C90) and the oscillation frequency of QPO for $r_s = 50$ is around 6Hz as seen in GRS1915+105 (public RXTE archival data of May 26th, 1997). Our $t_{b-b} = 42.5$s is encouraging, since on that day, bursts did repeat with $\nu_{burst} \sim 0.01$Hz. However, the system need not remain steady at these frequencies due to non-linear processes such as recycling a part of the wind back to the accretion disk. Systematic feeding of matter would raise the accretion rate, decreasing the cooling time and increasing the QPO frequency. Similarly, if the specific angular momentum remains higher, cool flow may take longer time to be drained, to the extent that the flow may like to stay for a long time in the burst phase in a soft state. When the Keplerian rate is actually increased in the inflow (due to a rise in viscosity at the outer edge of the disk, say), shocks would permanently cool down and outflows would be gradually turned off.

If the shock strength is on the higher side ($R > 2.5$), $f_0 \gg 1$, the location of the sonic point r_c increases linearly with f_0, but the average density $\langle\rho\rangle$ decreases exponentially (Eq. (13)). So, $t_{b-b} \sim \infty$ and the object will remain in the hard state. In this phase, hydrodynamic outflow may form continuously as the rate $R_{\dot{m}}$ does not go to zero. The QPO frequency ν_{QPO} is still determined by the infall time scale from the shock location. (This is true if the cooling time and the infall time become comparable so that QPO forms in the first place.) Only when the Keplerian rate is intrinsically increased due to, say, a rise of viscosity at the outer boundary of the disk, does the shock get softened and the t_{b-b} starts getting smaller within observable resolution. Thus, in this scenario, in the pure soft state, $R \to 1$ and in the pure hard state (with possible QPO) $R \to 4$–7. In between there is a possibility of having both soft (flaring) and

hard states (including QPO) switching in tens to hundreds of seconds with periodicity of t_{b-b}.

Since $t_{(b-b)} \propto r_s^2$ and $\nu_{QPO} \propto r_s^{-3/2}$ (Molteni et al. 1996), it is expected that $t_{(b-b)} \propto \nu_{QPO}^{-2}$ and since $\nu_{burst} \sim 1/t_{(b-b)}$, it is expected that $\nu_{burst} \propto \nu_{QPO}^2$. Details are presented in Chakrabarti (1990).

4. Concluding remarks

Although outflows are common in many astrophysical systems which include compact objects such as black holes and neutron stars, it was difficult to compute the outflow rates since these objects do not have any intrinsic atmospheres and outflowing matter has to come out from the inflow only. We showed in the present paper that, assuming the formation of a dense region around these objects (as provided by a centrifugal barrier, for instance), it is possible to obtain the outflow rate in a compact form with an assumption of isothermality of the outflow at least up to the sonic point and the ratio thus obtained seems to be quite reasonable. Computation of the outflow rate with a non-isothermal outflow explicitly depends on several flow parameters. Our primary goal in this paper was to obtain the rate as a function of the compression ratio of the gas and the geometric quantities. We show that for a given inflow/outflow configuration, the outflow rate shows a peak as the shock-compression is increased. We do not concern ourselves with the collimation mechanism. Since observed jets are generally hollow, they must be externally supported (either by ambient medium pressure or by magnetic hoop stress). This is assumed here for simplicity. Our assumption of isothermality of the wind till the sonic point is based on 'experience' borrowed from stellar physics. Momentum deposition from the hot photons from the dense cloud, or magnetic heating may or may not isothermalize the expanding outflow, depending on accretion rates and covering factors. However, it is clear that since the solid angle at which photons shine on electrons is close to 4π (as in a narrow funnel wall), and since the number of electrons per photon is much smaller in a compact region, it may be easier to maintain the isothermality close to a black hole than near a stellar surface.

The centrifugal-pressure-supported region that may be present was found to be very useful in explaining the soft and the hard states (CT95), rough agreement with power-law slopes in soft states (CT95) as well as the amplitude and frequency of QPO (C96) in black hole candidates. Therefore, our reasonable estimate of the outflow rate from these considerations further supports the view that such regions may be common around compact objects. Particularly interesting is the fact that since the wind here is thermally driven, the outflow ratio is higher for hotter gas, that is, for a low accretion rate. It is obvious that the non-magnetized neutron stars should also have the same dense region we discussed here and all the considerations mentioned here would be equally applicable.

It is to be noted that although the existence of outflows is well known, their rates are not. The only definite candidate whose outflow rate is known with any certainty is probably SS433 whose mass outflow rate was estimated to be

$\dot{M}_{out} \gtrsim 1.6 \times 10^{-6} f^{-1} n_{13}^{-1} D_5^2 M_\odot$ yr^{-1} (Watson et al. 1986), where f is the volume filling factor, n_{13} is the electron density n_e in units of 10^{13} cm^{-3}, D_5 is the distance of SS433 in units of 5kpc. Considering a central black hole of mass $10 M_\odot$, the Eddington rate is $\dot{M}_{Ed} \sim 0.2 \times 10^{-7} M_\odot$ yr^{-1} and assuming an efficiency of conversion of rest mass into gravitational energy $\eta \sim 0.06$, the critical rate would be roughly $\dot{M}_{crit} = \dot{M}_{Ed}/\eta \sim 3.2 \times 10^{-7} M_\odot$ yr^{-1}. Thus, in order to produce the outflow rate mentioned above even with our highest possible estimated $R_{\dot{m}} \sim 0.4$ (see Fig. 2a), one must have $\dot{M}_{in} \sim 12.5 \dot{M}_{crit}$ which is very high indeed. One possible reason why the above rate might have been over-estimated would be that below 10^{12}cm from the central mass (Watson et al. 1986), $n_{13} \gg 1$ because of the existence of the dense region at the base of the outflow.

We also discussed the effect of outflows on the spectral states and QPO properties of black hole candidates, such as GRS1915+105. The presence of QPO in the hard state, and periodic outbursts in intermediate state, generally absence of QPO in soft states, switching of states from hard to soft states in few seconds (free fall time rather than infall time) due to the presence of sub-Keplerian matter are generally understood in this scenario. Since the processes are highly non-linear, more detailed studies are necessary, but we believe that outflows play a major role in explaining these properties. In any case, qualitative agreement of the time scales decidedly prove that sub-Keplerian flows exist in a black hole accretion. Similar formation of outflows in neutron stars also should explain QPOs, especially that in neutron stars there could be *two* shocks (one near the hard surface and the other at a similar distance as the shock around a black hole; C90, C96). The only difference is that in neutron stars, the magnetic axis could be non-aligned with respect to the spin axis, and the outflows on either side of the disk would have an opposite effect in splitting the QPO frequency due to the Coriolis force as suggested in other contexts (Titarchuk et al. 1998).

In numerical simulations the ratio of the outflow and inflow has been computed on several occasions (Eggum et al. 1985; Molteni et al. 1994). Eggum et al. (1985) found the ratio to be $R_{\dot{m}} \sim 0.004$ for a radiation pressure dominated flow. This is generally comparable with what we found above (Eq. (19a)). In Molteni et al. (1994) the centrifugally driven outflowing wind generated a ratio of $R_{\dot{m}} \sim 0.1$. Here, the angular momentum was present in both inflow as well as outflow, and the shock was not very strong. Thus, the result is again comparable with what we find here. On the other hand, when the angular momentum is very high, it was seen that the outflow rate becomes comparable to the inflow rate. In these simulations, it was seen that the disk mass changes dramatically, and occasionally evacuating the disk also. This is possible if Θ_{out} is very large as in our present model.

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