

Research Note

Mean free path of solar electrons and nucleons

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Abstract. The quasilinear scattering mean free path of solar flare electrons and nucleons is calculated for interplanetary plasma wave turbulence consisting of a mixture of slab shear Alfvén waves and isotropic fast magnetosonic waves. It is shown that for well-sampled electron and nucleon mean free paths over a wide and overlapping energy range the underlying frequency power spectra of left-handed and right-handed slab Alfvén waves can be uniquely determined. Moreover, we demonstrate that the flatness and magnitude problems of quasilinear mean free paths of solar flare particles can be resolved if the interplanetary slab Alfvén magnetic power spectrum varies in wavenumber as $\propto |k|^{-s}$ with $s = 5/3$ below $|k| \leq k_c$ and $s = 3$ above $|k| > k_c$ where k_c is the inverse of the ion skin length.

Key words: plasmas – turbulence – Sun: flares – Sun: particle emission

1. Introduction

Cosmic ray particles in space plasmas are confined and accelerated by resonant interactions in weakly random electromagnetic fields. In the presence of low-frequency magnetohydrodynamic plasma waves, whose magnetic field component is much larger than their electric field component, the particle's phase space distribution function adjusts rapidly to a quasi-equilibrium through pitch-angle diffusion, which is close to the isotropic distribution. The isotropic part of the phase space distribution function $F(z, p, t)$ obeys the *diffusion-convection-equation* including diffusion and convection terms both in momentum (p) and position (z) space, where the space coordinate z is parallel to the uniform background magnetic field \mathbf{B}_0 .

Of particular interest is the parallel spatial diffusion coefficient $\kappa = v\lambda/3$ which conventionally is expressed in terms of the mean free path λ and particle speed v . Within quasilinear theory the mean free path results from the pitch-angle ($\mu = p_{\parallel}/p$) average of the inverse of the pitch-angle Fokker-Planck coefficient $D_{\mu\mu}$ as (Jokipii 1966, Hasselmann & Wibberenz 1968, Earl 1974)

$$\lambda = \frac{3v}{8} \int_{-1}^1 d\mu \frac{(1 - \mu^2)^2}{D_{\mu\mu}(\mu)}. \quad (1)$$

Schlickeiser & Miller (1997, 1998 – hereafter referred to as SM) have calculated the quasilinear Fokker-Planck coefficient $D_{\mu\mu}$ in a turbulent electromagnetic field consisting of a mixture of slab shear Alfvén waves and isotropic fast magnetosonic waves in a low β -plasma. They discussed possible new solutions to the well-known “flatness” and “magnitude” problems (Palmer 1982, Bieber et al. 1994) of the quasilinear mean free path. Here we extend this work and derive the interplanetary plasma turbulence conditions that allow the resolution of the mean free path discrepancy.

2. Quasilinear mean free path in mixed plasma wave turbulence

According to SM the Fokker-Planck coefficient $D_{\mu\mu}$ is the sum of contributions from transit-time damping (T) and gyroresonant interactions with the shear Alfvén (A) and fast mode (F) branch. At particle pitch-angles outside the interval $|\mu| \geq \epsilon = V_A/v$ transit-time damping provides the dominant contribution to the pitch-angle scattering, so that the value of the mean free path is primarily fixed by the small but finite scattering due to gyroresonant interactions in the interval $|\mu| < \epsilon$. It has been noted by SM that in this interval the gyroresonance fast mode contribution is much smaller than the shear wave contribution for comparable total wave intensities $(\delta B)_A^2$ and $(\delta B)_F^2$. Consequently, we may approximate the quasilinear mean free path (1) as

$$\lambda \simeq \frac{3v}{8} \int_{-\epsilon}^{\epsilon} d\mu (1 - \mu^2)^2 [D_{\mu\mu}^A(\mu)]^{-1} \simeq \frac{3v\epsilon}{4D_{\mu\mu}^A(\mu=0)}, \quad (2)$$

where we approximated $D_{\mu\mu}^A(|\mu| < \epsilon) \simeq D_{\mu\mu}^A(\mu=0)$. The quasilinear Fokker-Planck coefficient of a cosmic ray particle of gyrofrequency $\Omega = \Omega_0/\gamma$ for the parallel propagating undamped slab Alfvén wave branch is related to the one-dimensional wave number power spectrum of magnetic fluctuations $I(k)$ as

$$D_{\mu\mu}^A(\mu) = \frac{\pi\Omega^2(1 - \mu^2)}{2B_0^2}$$

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$$\times \int_{-\infty}^{\infty} dk I(k) \delta(\omega(k) - vk\mu - \Omega), \quad (3)$$

where we use the notation $\omega > 0$ and $\omega < 0$ to refer to left-handed and right-handed circularly polarised plasma waves, respectively. Note that the non-relativistic gyrofrequency is positive for nucleons and negative for electrons. For $\mu = 0$ we immediately obtain the frequency resonance

$$\begin{aligned} D_{\mu\mu}^A(\mu = 0) &= \frac{\pi\Omega^2}{2B_0^2} \int_{-\infty}^{\infty} dk I(k) \delta(\omega(k) - \Omega) \\ &= \frac{\pi\Omega^2}{2B_0^2} \int_{-\infty}^{\infty} d\omega J(\omega) \delta(\omega - \Omega) \\ &= \frac{\pi\Omega^2 J(\Omega)}{2B_0^2}, \end{aligned} \quad (4)$$

where we use the relation

$$I(k)dk = J(\omega)d\omega \quad (5)$$

between the frequency ($J(\omega)$) and wavenumber ($I(k)$) power spectra.

Inserting Eq. (4) into Eq. (2) we obtain for the mean free path

$$\lambda(\gamma) \simeq \frac{L_0}{\Omega_0^2} \frac{\gamma^2}{J\left(\frac{\Omega_0}{\gamma}\right)}, \quad (6)$$

where the constant L_0 is given by

$$L_0 \equiv \frac{3V_A B_0^2}{2\pi}. \quad (7)$$

The slab Alfvén wave branch has frequencies $|\omega| \leq |\Omega_{0,e}|$ and the dispersion relation

$$\omega = V_A k \left[\pm \sqrt{1 + \frac{k^2}{4k_c^2}} - \frac{k}{2k_c} \right], \quad (8)$$

where $k_c = \Omega_{0,p}/V_A = (c/\omega_{p,p})^{-1}$ is the inverse ion skin length and $\omega_{p,p}$ the proton plasma frequency. The dispersion relation (8) comprises the right-handed polarised Alfvén-Whistler branch at negative frequencies $\Omega_{0,e} < \omega < 0$ and the left-handed polarised Alfvén-ion-cyclotron branch at positive frequencies $0 < \omega < \Omega_{0,p}$, whereas no waves exist in the positive frequency range $\Omega_{0,p} < \omega < |\Omega_{0,e}|$. Solar electrons with $\mu = 0$ only interact with right-handed polarised waves, and Eq. (6) yields

$$\lambda_e(\gamma) \simeq L_0 \frac{\gamma^2}{\Omega_{0,e}^2} \frac{1}{J\left(\frac{\Omega_{0,e}}{\gamma}\right)}. \quad (9)$$

Alternatively, solar flare nucleons with $\mu = 0$ interact only with left-handed polarised waves, and Eq. (6) yields

$$\lambda_p(\gamma) \simeq L_0 \frac{\gamma^2}{\Omega_{0,p}^2} \frac{1}{J\left(\frac{\Omega_{0,p}}{\gamma}\right)}. \quad (10)$$

Obviously, using the frequency resonance condition (see Eq. (4)) $\omega = \Omega$, we can invert Eqs. (9) and (10) and determine

the frequency power spectra of the slab plasma waves for known electron and proton mean free paths:

(1) For the frequency power spectrum of right-handed ($\omega < 0$) polarised waves we find

$$J_{\text{RH}}(\omega) = \frac{L_0}{\omega^2} \left[\lambda_e \left(\frac{\Omega_{0,e}}{\omega} \right) \right]^{-1}, \quad (11)$$

which will lie in the Whistler frequency range because measured solar flare electrons have Lorentz factors well below 1836.

(2) In the case of protons we obtain for the left-handed ($\omega > 0$) polarised waves

$$J_{\text{LH}}(\omega) = \frac{L_0}{\omega^2} \left[\lambda_p \left(\frac{\Omega_{0,p}}{\omega} \right) \right]^{-1} \quad (12)$$

which lies in the Alfvénic frequency range because $\Omega_{0,p}/\gamma < \Omega_{0,p}$.

For well-sampled electron and proton mean free paths over a wide and overlapping energy range we may thus calculate the underlying frequency power spectrum of the slab plasma waves. Unfortunately, existing measurements of the particle mean free paths (Dröge 1994) do not have this quality.

3. Plasma turbulence spectra to resolve the mean free path discrepancy

Turbulence measurements in the interstellar medium (e.g. Dröge et al. 1993) suggest a Kolmogorov-type frequency power spectrum of the form

$$J(\omega) = J_0 |\omega|^{-s}, \quad (13)$$

with constants J_0 and s . According to Eqs. (9) and (10) we then obtain

$$\lambda_p(\gamma) \simeq \frac{L_0}{J_0} \left(\frac{\gamma}{\Omega_{0,p}} \right)^{2-s}, \quad \lambda_e(\gamma) \simeq \frac{L_0}{J_0} \left(\frac{\gamma}{\Omega_{0,e}} \right)^{2-s}. \quad (14)$$

First, we note that for *any* value of s this yields a constant mean free path of protons

$$\begin{aligned} \lambda_p &\simeq \lambda_0 = \frac{L_0 \Omega_{0,p}^{s-2}}{J_0} \\ &= \frac{24}{s-1} \left(\frac{B_0}{\delta B_A} \right)^2 \frac{c}{\omega_{p,p}} \left(\frac{\Omega_{0,p}}{\omega_0} \right)^{s-1} \end{aligned} \quad (15)$$

at non-relativistic kinetic energies because

$$\gamma = 1 + \frac{E_{\text{kin}}}{mc^2} \simeq 1. \quad (16)$$

In Eq. (15) we use

$$\frac{(\delta B)_A^2}{8\pi} = 2 \int_{\omega_0}^{\infty} d\omega J(\omega) \simeq \frac{2J_0}{s-1} \omega_0^{1-s}, \quad (17)$$

where ω_0 denotes the lowest frequency of the plasma waves.

Qualitatively, using $s = 5/3$, an interplanetary magnetic field strength of $B_0 = 10^{-4} B_{-4}$ G, an ion density of $n_0 = 10 n_{10}$ cm^{-3} (e.g. Benz 1993, Table 4.1) and $\omega_0 = 10^{-4} \omega_{-4} \text{ s}^{-1}$

Table 1. Magnetic field data and proton mean free path for three solar events

event	$J_0/G^2 Hz^{2/3}$	λ_0 / AU	λ_{obs} / AU
1977 Nov. 22	$2.0 \cdot 10^{-12}$	0.11	0.06–0.13
1977 Dec. 27	$3.0 \cdot 10^{-13}$	0.74	0.10–0.18
1978 Apr. 11	$1.5 \cdot 10^{-11}$	0.015	0.02–0.03

for the lowest plasma wave frequency (Belcher & Davis 1971, March & Tu 1990) we obtain for the constant mean free path according to Eq. (15)

$$\lambda_0 = 0.074 \left[\frac{(B_0/\delta B_A)^2}{10} \right] B_{-4}^{2/3} n_1^{-1/2} \omega_{-4}^{-2/3} AU, \quad (18)$$

which agrees well with measured proton and electron mean free paths (Dröge et al. 1993, Bieber et al. 1994).

3.1. Quantitative comparison with observations

For eight solar particle events observed on board the Helios 1 and 2 spacecraft magnetic field data from the Helios fluxgate magnetometer are available (Wanner & Wibberenz 1993, Dröge et al. 1993). The spectral slopes of the power spectra are quite similar and at low frequencies agree well with the Kolmogorov spectrum $J(\omega) \propto \omega^{-5/3}$. However, the absolute level of the turbulence (i.e. J_0 in Eq. (13)) varies considerably from event to event. No spectral data at frequencies above 0.06 Hz are available for these events due to the limited time resolution of the Helios magnetometer. For three of these eight events, listed in Table 1, we use the published power spectra (Dröge et al. 1993) for a quantitative comparison of our results.

Combining Eqs. (7) and (15) we obtain

$$\lambda_0 = 2.2 \cdot 10^{-13} J_0^{-1} \frac{B_{-4}^{8/3}}{n_1^{1/2}} AU, \quad (19)$$

where we used $V_A = 6.7 \cdot 10^6 B_{-4} n_1^{-1/2} \text{ cm s}^{-1}$. With the observed turbulence levels J_0 we calculate in Table 1 λ_0 according to this equation assuming that the unobserved values of $B_{-4} = 1$ and $n_1 = 1$ are unity. As one can see from Table 1 there is excellent agreement between the calculated and observed mean free paths for the 1977 Nov. 22 and the 1978 Apr. 11 event, whereas for the 1977 Dec. 27 event the calculated mean free path is about a factor 6 larger than the observed value. Of course, this discrepancy can be removed by adopting a six times smaller value for $B_{-4}^{8/3} n_1^{-1/2}$.

3.2. Mean free path of electrons

Using $J_2|\omega|^{-2} = J_0|\omega|^{-s}$ at $\omega = -\Omega_{0,p}$, a constant mean free path of electrons,

$$\lambda_e = \frac{L_0}{J_2} = \frac{L_0}{J_0} \Omega_{0,p}^{s-2}, \quad (20)$$

equal to the non-relativistic proton value (15), results if the special value $s = 2$ is taken for the frequency power spectrum in the *Whistler* frequency range. In this frequency range the full dispersion relation (8) can be approximated as

$$\omega \simeq -V_A k^2 / k_c, \quad (21)$$

so that relation (5) yields the corresponding wavenumber power spectrum

$$I(k) = J(\omega) \left| \frac{d\omega}{dk} \right| = I_0 k^{-3}, \quad (22)$$

with $I_0 = 2J_0 k_c / V_A$. Note that this special form of the high wavenumber power spectrum is the limit of allowable dissipation range spectra in order that the mean square curl of the turbulent magnetic field be finite (Bieber et al. 1990, 1994).

We noted before that no spectral data at frequencies above 0.06 Hz are available for the eight solar events. Earlier magnetic field observations with higher time resolution (Coroniti et al. 1982, Denskat et al. 1983) indicated a steepening of the magnetic field power spectra at a frequency of about ~ 1 Hz in the spacecraft frame of reference. These observations are in accord with our assumptions here.

To conclude: the flatness and magnitude problems of quasi-linear mean free paths of solar flare particles can be resolved if the interplanetary slab Alfvén wavenumber magnetic power spectrum varies $\propto |k|^{-s}$ below $|k| \leq k_c$ with $s = 5/3$, and $\propto |k|^{-3}$ above $|k| > k_c$.

References

- Belcher J.W., Davis L., 1971, JGR 76, 3534
Benz A.O., 1993, Plasma Astrophysics. Kluwer, Dordrecht
Bieber J.W., Smith C.W., Matthaeus W.H., 1990, ApJ 334, 470
Bieber J.W., Matthaeus W.H., Smith C.W., et al., 1994, ApJ 420, 294
Coroniti F.V., Kennel C.F., Scarf F.L., Smith E.J., 1982, JGR 87, 6029
Denskat K.U., Beinroth H.J., Neubauer F.M., 1983, J. Geophys. 54, 60
Dröge W., 1994, In: Leahy D.A., Hicks R.B., Venkatesan D. (eds.) Proc. 23rd ICRC, World Scientific, Singapore, p. 185
Dröge W., Achatz U., Wanner W., Schlickeiser R., Wibberenz G., 1993, ApJ 407, L95
Earl J.A., 1974, ApJ 193, 231
Hasselmann K., Wibberenz G., 1968, Z. Geophys. 34, 353
Jokipii J.R., 1966, ApJ 146, 480
March E., Tu C.-Y., 1990, JGR 95, 8211
Palmer I.D., 1982, Rev. Geophys. 20, 335
Schlickeiser R., Miller J.A., 1997, Proc. 25th ICRC, Durban, Vol. 1, p. 237
Schlickeiser R., Miller J.A., 1998, ApJ 492, 352
Wanner W., Wibberenz G., 1993, JGR 98, 3513