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Uncertainty of pulsar time scale due to the gravitational time delay of intervening stars and MACHOs

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Abstract. As a cause of possible uncertainty of the pulsar time scale, we investigated the gravitational time delay due to the motion of the intervening stars and MACHOs. We calculated the amplitudes of cubic, quartic and quintic trends in the residual of the times of arrival (TOA) of the pulse from pulsar due to gravitational time delay. It is shown that the cubic trend becomes dominant when the timing measurement accuracy is relatively high, say higher than 10 micro second at the case of the intervening star's mass is 1 $M_{\odot}$. The optical depth of three trends are shown as a function of TOA residual. The optical depth for detecting the cubic trend is approximately proportional to the 2/3 th power of the mass over the timing measurement accuracy, and to the square of the observational period. Typical order of this optical depth is 0.1 for a pulsar of a few kpc distance and observed over 10 years with the timing measurement accuracy of 10 ns.

Key words: gravitation – relativity – time – stars: pulsars: general – stars: pulsars: individual: PSR 1937+21

1. Introduction

It is well known that the pulse period of the millisecond pulsars is very stable. Some of the millisecond pulsars such as PSR B1937 +21 and PSR B1855 +09 have been observed over a decade and the times of arrival (TOAs) of their pulses have been measured very accurately (Kaspi et al. 1994). They have shown that the stabilities of their pulses are comparable to those of atomic clocks on the Earth. By using these accurate observations, many authors had pointed out the possibility of the construction of Pulsar Time Scale (I'llin et al. 1986; Foster & Backer 1990; Guinot & Pettit 1991; Petit & Tavella 1996; Matsakis et al. 1997). On the other hand, in the construction of the time scale, it is also pointed out that we have to be careful of many effects which might affect the stability of the pulse period of the pulsars (Guinot & Pettit 1991; Petit & Tavella 1996; Petit 1995). In the case of millisecond pulsars, the residuals are mainly characterized by random jitter and slow variations. The analysis procedure of the pulsar TOA is well established (Backer & Hellings 1986; Tayler & Weisberg 1989). Periodic effects and up to quadratic trends with respect to time are solved and the set of post-fit residuals of TOA are obtained, namely the differences between the measured TOA's and those expected according to a model. Cubic and higher trends in time are expected to be very small. However, in the set of post-fit residuals, some systematic trends are often seen. In many observed pulsars, much larger breaking indices than expected by dipole radiation theory and some strange long term trend are still to be explained (Arzoumanian et al. 1994; Kaspi et al. 1994). They are considered as the presence of unmodeled effects. Such effects are classified into three categories, i.e, pulsar itself, pulsar system in the case of binary and the effects in the propagation of the signal. As one of the possible effects in the propagation, we have considered the gravitational time delay of the pulse due to the gravitational field of the stars and the Massive Astrophysical Compact Halo Objects (MACHOs) between the pulsar and the observer (Ohnishi et al. 1995). In that paper, we investigated how to measure the mass of intervening stars and MACHOs using this delay. and the possibility of detecting this effect. Such a gravitational delay has also been discussed by some other authors (Larchenko & Doroshenko 1995; Walker 1996; Fargion & Conversano 1997).

Recently, we showed that the position of the quasars fluctuated in the micro-arcsecond level due to the gravitational field of the stars and MACHOs in our galaxy (Hosokawa et al. 1997). The situation would be the same for the timing of the signal that comes from pulsars in our galaxy.

In this paper, we discuss further aspects of the gravitational delay and formulate the TOA fluctuation due to the delay as the function of the measurement accuracy and the mass of the delay source as well as the other quantities we have discussed before. We assume here that the unexpected cubic and higher trends are due to the gravitational delay. As a method for analyzing the slow variation part of residuals, we consider its expansion by Legendre's polynomial series (Deeter 1984; Stinebring et al. 1991). In Sect. 2, we introduce the concept of Legendre's polynomial expansion and evaluate the maximum value of each polynomial in the residual of gravitational time delay. In Sect. 3, we derive the formula of the optical depth of the gravitational

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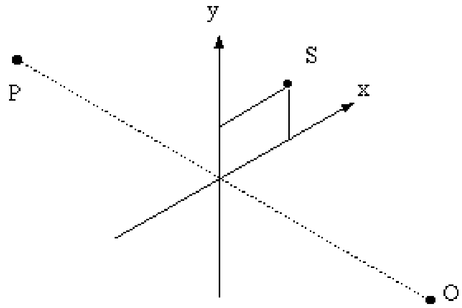

Fig. 1. The configuration of the pulsar, the delay source and the observer.

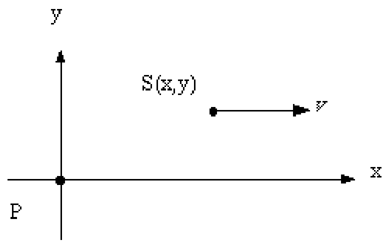

Fig. 2. The positions and the relative motion of the pulsar and the delay source in the delay plane.

delay. Using this formula, we estimated the optical depth for some cases study in Sect. 4.

2. Legendre expansion of gravitational delay

Masses close to the pulse path from the pulsar to the Earth will produce the gravitational time delay of the pulse TOA. Relative motion of such delay source to the pulse path causes the variation of the time delay. The configuration of the pulsar, the delay source, and the observer is shown in Fig. 1. For the description of this time delay, let us consider a tangent plane of the celestial sphere, like the lens plane of gravitational lensing (Schneider et al. 1992, p.31). Here we call this plane the delay plane. We take the coordinate system on the delay plane so that the projected position of the pulsar is at the origin and the position of the delay source is (x, y). We also chose the x-axis as the direction of the projected motion of the delay source on the delay plane. Remark that, in the previous paper (Ohnishi et al. 1995), we took the origin of the coordinate system on the delay plane as the projected position of the delay source. Here, we moved the origin to that of the pulsar. This is because this new convention makes the following analytic expression simpler.

The relative motion between the pulsar and the delay source is well approximated to be linear (Fig. 2). Therefore

$$x = v(t - t_0), \quad y = \text{constant}. \tag{1}$$

where v is a constant and t_0 is the epoch of the closest approach.

The gravitational time delay is expressed as a function of x and y (Ohnishi et al. 1995) as

$$\Delta t = constant - m f_{\rm d}(x, y). \tag{2}$$

Here

$$f_{\rm d}(x,y) \;=\; \log\left(1 + \frac{x^2}{y^2}\right), \tag{3}$$
$$m \;\equiv\; \frac{2GM}{c^3} = 9.751\,\mu s \times \frac{M}{M_\odot}.$$

and M is the mass of the delay source, G is the Newton's constant of gravitation, and c is the speed of light in vacuum, and $M_\odot$ is the mass of the Sun. Hereafter, we call $f_{\rm d}(x, y)$ as the characteristic function. One of the most notable features of this function is that it is independent of the distance between the observer and the delay source.

In order to expand this characteristic function by Legendre polynomials, we introduce a new variable X,

$$X = 2(t - t_{\rm e})/T, \quad -1 \le X \le 1 \tag{4}$$

where T is the observation period, $t_{\rm e}$ is the epoch of the mid point of the observation, and $(x_{\rm e}, y_{\rm e})$ is the position of the delay source at the epoch $t_{\rm e}$. Remark that $t_{\rm e}$ is not coincident with t_0 usually. Then the characteristic function is rewritten as

$$f_{\rm d}(X; x_{\rm e}, y_{\rm e}) = \log\left[1 + \left(x_{\rm e} + \frac{vT}{2}X\right)^2 / y_{\rm e}^2\right], \tag{5}$$

In the post-fit residual of the pulsar timing date, linear and quadratic trend are removed automatically in the fitting of P, $\dot{P}$. On the other hand, for stable millisecond pulsars, higher order trends are expected to be small. Since there are three dimensionless parameters m, $x_{\rm e}/vT$ and $y_{\rm e}/vT$, for the determination of them we have to obtain the amplitudes of at least three different polynomials. Note that we cannot separate vT from $x_{\rm e}$ nor $y_{\rm e}$ by observing only the time delay (Ohnishi et al. 1995). Therefore we consider the expansion up to the fifth order and neglect the sixth and higher order polynomials.

$$f_{\rm d}(X; x_{\rm e}, y_{\rm e}) \approx \sum_{n=0}^{5} c_n(x_{\rm e}, y_{\rm e}) P_n(X) \tag{6}$$

where $P_n(X)$ and $c_n(x_{\rm e}, y_{\rm e})$ are the Legendre polynomial of n-th order and its expansion coefficient, respectively. The polynomials up to the quadratic one are absorbed in the fitting of P and $\dot{P}$, and what we are interested in are those of cubic and higher orders. The explicit expression of those polynomial is as follows.

$$P_3(X) \;=\; (5X^3 - 3X)/2\,,$$
$$P_4(X) \;=\; (35X^4 - 30X^2 + 3)/8\,,$$
$$P_5(X) \;=\; (63X^5 - 70X^3 + 15X)/8\,. \tag{7}$$

We obtain the coefficients in Eq. (6) easily by comparing the coefficients of its Taylor expansion;

$$f_{\rm d}(X; x_{\rm e}, y_{\rm e}) = \sum_{n=0}^{5} \frac{1}{n!} f_{\rm d}^{(n)}(0; x_{\rm e}, y_{\rm e}) X^n + O(X^6) \tag{8}$$

The n'th derivative of the characteristic function with respect to X at $X = 0$, $f_{\rm d}^{(n)}(0; x_{\rm e}, y_{\rm e})$, are calculated as

$$f_{\rm d}^{(3)}(0; x_{\rm e}, y_{\rm e}) = \frac{-v^3 T^3}{2} F_3(x_{\rm e}, y_{\rm e})$$
$$f_{\rm d}^{(4)}(0; x_{\rm e}, y_{\rm e}) = \frac{3v^4 T^4}{4} F_4(x_{\rm e}, y_{\rm e})$$
$$f_{\rm d}^{(5)}(0; x_{\rm e}, y_{\rm e}) = \frac{-3v^5 T^5}{2} F_5(x_{\rm e}, y_{\rm e}) \quad (9)$$

where $F_n(x, y)$ $(n = 3, 4, 5)$ are

$$F_3(x, y) = \frac{x(x^2 - 3y^2)}{(x^2 + y^2)^3}$$
$$F_4(x, y) = \frac{(x^4 - 6x^2y^2 + y^4)}{(x^2 + y^2)^4}$$
$$F_5(x, y) = \frac{x(x^4 - 10x^2y^2 + 5y^4)}{(x^2 + y^2)^5} \quad (10)$$

It is worth noting that when we use the polar coordinates $x = r\cos\theta, y = r\sin\theta$, F_n are expressed simply as

$$F_n(r, \theta) = \frac{\cos n\theta}{r^n}. \quad (11)$$

By comparing Eq. (6) with Eq. (8), the coefficients c_n is obtained as

$$c_3 = \left(\frac{2}{5}\right)\frac{1}{3!} f_{\rm d}^{(3)}(0; x_e, y_e) = \frac{4}{15} F_3(x_{\rm e}, y_{\rm e}) \left(\frac{vT}{2}\right)^3$$
$$c_4 = \left(\frac{8}{35}\right)\frac{1}{4!} f_{\rm d}^{(4)}(0; x_e, y_e) = \frac{4}{35} F_4(x_{\rm e}, y_{\rm e}) \left(\frac{vT}{2}\right)^4$$
$$c_5 = \left(\frac{8}{63}\right)\frac{1}{5!} f_{\rm d}^{(5)}(0; x_e, y_e) = \frac{16}{315} F_5(x_{\rm e}, y_{\rm e}) \left(\frac{vT}{2}\right)^5 \quad (12)$$

Here we neglect the higher order terms and assume that the distance of the star from the line of sight to the pulsar r is much larger than $vT/2$, then $f^{(3)}(0) \gg f^{(5)}(0)$. Next, let us consider how much each component contribute to the residual. We evaluate them by the maximum value in the observation period for each polynomial. The maximum value of each Legendre's polynomial is that at the end point $X = \pm 1$, that is, $P_n(1) = 1$. Using this fact and the coefficients Eq. (12), we obtain the maximum value of the n-th order trend of the residual as

$$\Delta t_n = m c_n. \quad (13)$$

These equations show us clearly how the largest residual in each component depends on the position of the delay source at the epoch t_e on the delay plane, the velocity, the mass and the observation period.

3. Optical depth of gravitational delay

The optical depth of the gravitational time delay is important when we consider the observability that the delay makes a larger residual than the accuracy of TOA measurement. Let us consider the quantity $S_n(\Delta t)$, that is the area of the region in the delay plane where Δt_n is larger than a certain value Δt.

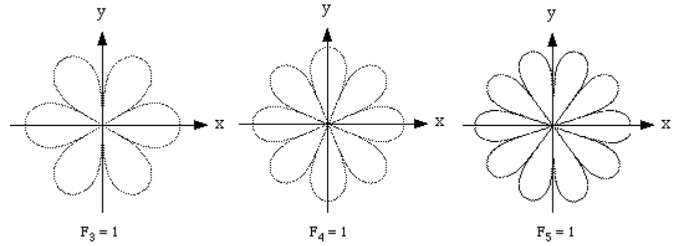

Fig. 3. The curve $|F_n| = 1$. The areas that give larger maximum residual than a certain value are proportional to these curves in the delay plane.

From Eq. (13) together with Eq. (10), we see that any position where the absolute values of $F_n(x_e, y_e)$ are the same will make the same Δt_n. Thus, when the position of the source (x_e, y_e) is located within a curve $|F_n| = const.$, the corresponding Δt is larger than Δt_n. The curves $|F_n| = 1$ are shown in Fig. 3. Let us denote the area inside the curve $|F_n| = 1$ by K_n. Numerically they are calculated as $K_3 = 2.24$, $K_4 = 2.40$, $K_5 = 2.51$. By means of the values K_n, $S_n(\Delta t)$ are expressed as

$$S_3(\Delta t) = K_3 \left(\frac{4m}{15\Delta t}\right)^{\frac{2}{3}} \left(\frac{vT}{2}\right)^2 = 0.91 \left(\frac{m}{\Delta t}\right)^{\frac{2}{3}} \left(\frac{vT}{2}\right)^2$$
$$S_4(\Delta t) = K_4 \left(\frac{4m}{35\Delta t}\right)^{\frac{1}{2}} \left(\frac{vT}{2}\right)^2 = 0.82 \left(\frac{m}{\Delta t}\right)^{\frac{1}{2}} \left(\frac{vT}{2}\right)^2$$
$$S_5(\Delta t) = K_5 \left(\frac{16m}{315\Delta t}\right)^{\frac{2}{5}} \left(\frac{vT}{2}\right)^2$$
$$= 0.75 \left(\frac{m}{\Delta t}\right)^{\frac{2}{5}} \left(\frac{vT}{2}\right)^2 \quad (14)$$

These equations show us that the areas of all the components are proportional to $(vT/2)^2$ but the dependence on $m/\Delta t$ is different from each other. As the increase of $m/\Delta t$, the cubic trend becomes dominant.

By means of the areas $S_n(\Delta t)$, the optical depth of the gravitational delay for the case that the maximum value of the n-th order residual is larger than Δt is expressed as

$$\tau_n(\Delta t) = \int_0^D N(z)\, S_n(\Delta t) {\rm d}z \quad (15)$$

where D is the distance of the pulsar, $N(z)$ is the number density of delay source at the distance z from the observer.

It should be noted that, if the mass density is constant and the number density of the delay source is inversely proportional to the average mass, the area $S_n(r)$ is proportional to the $(2/n)$-th power of the mass. In the case of the cubic term, the optical depth is proportional to the $(-1/3)$ th power of the average mass of the delay source. Therefore, under that condition, the delay source of the smaller average mass gives the larger optical depth of the gravitational time delay.

4. Case study

Let us estimate the typical value of the optical depth of the detectable variation of the pulsar TOA due to the gravitational time delay. As the delay source, two kinds of objects should be considered; the disk stars and MACHOs.

When a pulsar is close to the galactic plane and its distance from us is not so large, say, within a few kpc, the number densities can be regarded as constant, namely equal to the local densities of stars and MACHOs, respectively. If the velocity of a pulsar relative to the observer, v_p, is much larger than that of the nearby stars, the velocity of the stars projected onto the delay plane is

$$v(z) = \frac{z}{D} v_p \tag{16}$$

For a rough evaluation of this optical depth, we take $v_p = 400\,\mathrm{km\,s^{-1}}$ (Lyne & Lorimer 1994), T as 10 years, and the average mass of the disk stars as $0.2M_\odot$ which comes from the typical density of M-type dwarf, $0.1M_\odot\mathrm{pc}^{-3}$. In this case the optical depths of each order of trend are obtained easily. Here we take kpc as the unit of the distance D and ns as the unit of the maximum residual Δt.

$$\begin{aligned} \tau_{3,D}(\Delta t) &= 0.10D/(\Delta t)^{\frac{2}{3}}, \\ \tau_{4,D}(\Delta t) &= 0.026D/(\Delta t)^{\frac{1}{2}}, \\ \tau_{5,D}(\Delta t) &= 0.011D/(\Delta t)^{\frac{2}{5}}. \end{aligned} \tag{17}$$

If MACHOs whose transverse velocities are random with the dispersion of v_M exist, the average relative transverse velocity between them and the line of sight to the pulsar is expected to be

$$v(z) = \left\{ (\frac{z}{D} v_p)^2 + v_M^2 \right\}^{\frac{1}{2}} \tag{18}$$

By using this equation and supposing MACHOs as $0.1M_\odot$ average mass, $0.008M_\odot\mathrm{pc}^{-3}$ density and $v_M = 200\,km\,s^{-1}$, the corresponding optical depth due to MACHOs are,

$$\begin{aligned} \tau_{3,M}(\Delta t) &= 0.018D/(\Delta t)^{\frac{2}{3}}, \\ \tau_{4,M}(\Delta t) &= 0.0047D/(\Delta t)^{\frac{1}{2}}, \\ \tau_{5,M}(\Delta t) &= 0.0023D/(\Delta t)^{\frac{2}{5}}. \end{aligned} \tag{19}$$

These optical depths are shown in Fig. 4.

In the case that pulsars are far from us or from the galactic plane, their distribution in our galaxy should be taken into account. For example, PSR B1937+21 is located at $l = 58$, $b = 0$ with the proper motion of $\mu_\alpha = -0.13(mas/y), \mu_\delta = -0.46(mas/y)$ and the distance to that is estimated as 8 kpc (Kaspi et al. 1994). In such a case, the galactic rotation and the variation of the density in a galactic scale cannot be neglected. On the distribution and the rotation velocity of them, here we adopt the same model as that used in Hosokawa et al.(1997). According to the flat rotation model, the disk stars near PSR B1937+21 have the transverse velocity of about $200\,km\,s^{-1}$, i.e., $6mas/y$ proper motion with respect to the non-rotating celestial frame. Hence the transverse velocity of PSR B1937+21 relative to the disk stars is about $200\,km\,s^{-1}$. Using these values and model, the optical depth for PSR B1937+21 τ_3 due to the disk stars and MACHOs are obtained, respectively.

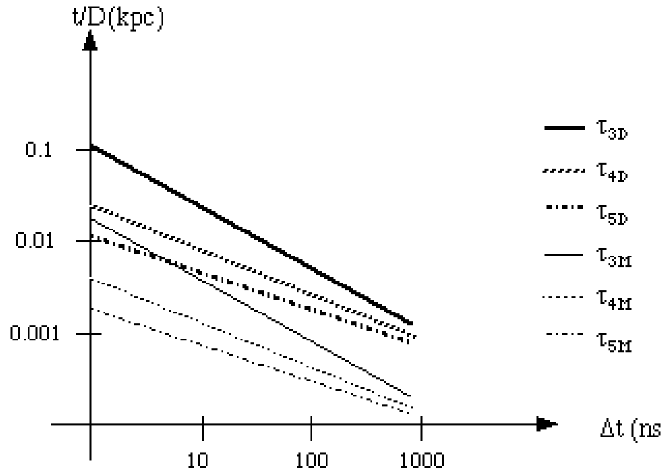

Fig. 4. Optical depths of Gravitational delay as the function of Δt normalized by the distance of Pulsar.

$$\begin{aligned} &\tau_{3,D}(\Delta t) = 0.20/(\Delta t)^{\frac{2}{3}}, \\ &\tau_{3,M}(\Delta t) = 0.08/(\Delta t)^{\frac{2}{3}}. \end{aligned} \tag{20}$$

These results show us that the probability Δt is larger than 1000 ns would be about 0.3 %.

5. Summary

We showed how the gravitational delay makes the n-th order trend in the post-fit residual of TOA. The optical depth of the delay is proportional to the square of the observation period and the relative velocity, and proportional to the $(2/n)$-th power of the ratio of the TOA measurement accuracy. As shown in Fig. 3, the characteristic shape of the equal maximum residual curve of each order is different. The quartic or quintic trend will contribute to the total optical depth in a small area that cannot be covered by the cubic trend. However, the cubic trend becomes more important than the higher order ones as the measurement accuracy becomes higher.

When we construct the pulsar time scale, many effects that make the trend in the post fit residual should be considered. The residual due to the gravitational delay discussed here is only one of them. Considering the distribution of stars and MACHOs known so far, the optical depth for this delay is not so large, as we have already shown and Walker pointed out (Ohnishi et al. 1995; Walker 1996). Even when the measurement accuracy reaches to 10 ns it would be in the order of 10^{-1} for the pulsar of a few kpc distant from us observed over ten years. That for the maximum residual of 1 microsecond would be a few thousandth. However, the residual due to the gravitational delay have some clear features such as the frequency independence, very long term trend and smooth variation of the residual. Therefore, by using these features it would be possible to separate the effect due to the gravitational delay from many other effects, such as the effects due to interstellar medium or the gravitational wave of the frequency higher than $10^{-8} Hz$.

Pulsar timing data would be useful to make the time scale stable in the long term, though it would be difficult to construct the time scale by themselves alone (Guinot & Petit 1991). In such a case, the evaluation of the long term effects like the gravitational delay is important. Recently, the accuracy and the stability of the atomic time scale on Earth reaches the order of 10^{-15} (Petit 1997). In order that the pulsar time should contribute to the estimation of long term stability in such high level with a few years' observation, the accuracy of TOA is required to the order of 100 ns or higher. Rapid improvement of recent measurement technology will make it possible to measure the TOA of some millisecond pulsars with such accuracy.

As the accumulation of the observation period and the improvement of timing measurement accuracy, together with the elucidation of the nature of the timing residual, the gravitational delay due to the intervening stars will become important to bring out the true stability of the pulsar time scale and to construct the stable time scale together with the atomic clocks on Earth. Also it would be useful for investigating the distribution of the massive object between pulsars and us.

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