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Coronal potential magnetic fields from photospheric sources with finite width

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Abstract. Finite width photospheric sources are used to generate coronal potential magnetic field configurations. The prescription of a suitable distribution of the magnetic flux function within such regions allows to solve the Grad-Shafranov equation and, using the superposition principle, to obtain magnetic field configurations related to arbitrary combinations of photospheric sources and sinks. Following this approach, we have focused our attention on bipolar and quadrupolar magnetic configurations in a background horizontal magnetic field, which creates the conditions for complex magnetic field topologies with magnetic X-points and local dips (minima). These configurations, with infinitely thin and point sources, have been previously invoked (Priest et al. 1994; 1996) to explain the process of photospheric flux cancellation (cancelling magnetic features) and prominence formation from photospheric material. We have investigated how the different parameters of the model (i.e. source width and magnetic strength) influence the magnetic field topology and have compared our results to previous ones.

Key words: Magnetohydrodynamics (MHD) – methods: analytical – Sun: corona – Sun: magnetic fields

1. Introduction

EUV and soft X-ray observations of the solar corona, made by Skylab more than two decades ago, pointed out that it is structured by the magnetic field. This structuring has been strikingly confirmed by the spectacular soft X-ray pictures taken by Yohkoh spacecraft during recent years.

Coronal magnetic field lines are rooted in the photosphere in the form of concentrated, isolated flux tubes and, for such reason, photospheric conditions are extremely important in order to determine the coronal field. One simple but commonly used approach to model coronal magnetic fields is based on infinitely long, structureless flux sources placed on the photosphere. The coronal field is then invariant along the axis of the photospheric sources and is represented by the field lines in a vertical plane perpendicular to them. This approach has allowed to model the quasi static evolution of cancelling magnetic features (Priest et al. 1994) and magnetic configurations presumed to provide mass and magnetic flux to prominences (Priest et al. 1996; Mackay & Priest 1996). Instead of this infinitely thin (line) sources we here incorporate extense sources and investigate the effect of their width on the above processes.

To obtain the coronal magnetic configuration we solve the Grad-Shafranov equation after prescribing a suitable distribution of magnetic flux within the photospheric magnetic source. Then, using the superposition principle, we generate coronal potential magnetic field configurations related to an arbitrary combination of localised sources and sinks of magnetic field lines on the photosphere.

The analytical expressions for the computation of the coronal magnetic structure from photospheric boundary conditions are derived in Sects. 2 and 3. The particular photospheric magnetic elements used in this work and the associated coronal flux function and magnetic field components are described in Sect. 4. There results are applied to the problem of cancelling magnetic features and to the formation and magnetic flux replenishing of quiescent prominences in Sects. 5 and 6, respectively. Finally, in Sect. 7 conclusions are drawn.

2. Potential magnetic field configurations

The coronal magnetic field structure is treated in Cartesian coordinates with the z-axis in the vertical direction. All physical quantities are assumed stationary and y-invariant, so they depend on the variables x and z only.

Potential magnetic fields are characterised by

$$\nabla \times \boldsymbol{B} = \boldsymbol{0}$$

and, in what follows, we shall assume $B_y = 0$, i.e. we consider a y-invariant, potential magnetic field contained in the x, z plane. Then,

$$\boldsymbol{B} = \nabla A \times \hat{\mathbf{e}}_y ,$$

i.e.

$$B_x = -\frac{\partial A}{\partial z}, \qquad B_z = \frac{\partial A}{\partial x}. \tag{1}$$

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This is just a particular case of the more general force-free solution and the expression for $A(x,z)$ can be obtained from the Grad-Shafranov equation with $B_y = 0$,

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2} = 0. \quad (2)$$

To determine the flux function in the corona ($z > 0$) one must integrate this partial differential equation, imposing some boundary conditions. We prescribe the value of the flux function at the photospheric level, $A(x, z = 0)$, and assume that A does not diverge as $z \to \infty$.

3. Boundary conditions and solutions

To solve Eq. (2), we first Fourier transform it in x, which gives

$$\frac{d^2 A_{(k)}}{dz^2} - k^2 A_{(k)} = 0, \quad (3)$$

where the Fourier transform $A_{(k)} = A_{(k)}(z)$ is defined by

$$A_{(k)}(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(x,z) e^{-\mathrm{i}kx} dx.$$

The solution to Eq. (3) that satisfies the mentioned boundary conditions now follows as

$$A_{(k)}(z) = A_{(k)}(0) \mathrm{e}^{-kz}, \quad (4)$$

where $A_{(k)}(0)$ is the Fourier transform of the boundary condition $A(x,0)$,

$$A_{(k)}(0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(x,0) e^{-\mathrm{i}kx} dx. \quad (5)$$

Finally, $A(x,z)$ is obtained by applying the inverse Fourier transform to $A_{(k)}(z)$ given by Eq. (4),

$$A(x,z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A_{(k)}(z) e^{\mathrm{i}kx} dk. \quad (6)$$

The magnetic field components B_x and B_z easily follow from this expression and Eq. (1),

$$B_x(x,z) = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{d}{dz} A_{(k)}(z) e^{\mathrm{i}kx} dk,$$
$$B_z(x,z) = \frac{\mathrm{i}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A_{(k)}(z) e^{\mathrm{i}kx} k dk.$$

4. Magnetic fields from localised sources

The photospheric distribution of the flux function $A(x,0)$ has so far been arbitrary. In what follows, we shall introduce the basic element of this paper, a finite width, y-invariant source of magnetic field, which can be used to generalise the infinitely thin line current source used by other authors (e.g. Priest et al. 1994; Mackay & Priest 1996).

Suitable combinations of unipolar sources can then be used to model more complex photospheric structures and the related coronal magnetic field configurations.

4.1. Unipolar magnetic region

We model a unipolar magnetic region that is localised around $x = a$ by a boundary condition $A(x,0)$ of the following form

$$A(x,0) = A_0 \tanh\left(\frac{x-a}{L}\right). \quad (7)$$

Here, A_0 and L are free constants defining the strength and the linear extent of the magnetic region, respectively. The sign of A_0 determines whether the region is a local source ($A_0 > 0$) or sink ($A_0 < 0$) of coronal magnetic field lines.

Substituting Eq. (7) into Eq. (5), we first get the expression for $A_{(k)}(0)$,

$$A_{(k)}(0) = -\mathrm{i}\frac{\sqrt{2\pi}}{2} \frac{A_0 L}{\sinh\left(\frac{\pi}{2}kL\right)} \mathrm{e}^{-\mathrm{i}ka}.$$

Next, Eqs. (4) and (6) yield the related solution for the flux function $A(x,z)$,

$$A(x,z) = A_0 L \int_0^{\infty} \frac{\sin[k(x-a)]}{\sinh\left(\frac{\pi}{2}kL\right)} \mathrm{e}^{-kz} dk, \quad (8)$$

which can also be expressed (Gradshteyn & Ryzhik 1980) in series form as

$$A(x,z) = \sum_{n=1}^{\infty} \frac{8A_0(x-a)/L}{[2(x-a)/L)]^2 + [2z/L + (2n-1)\pi]^2}, \quad (9)$$

or, using the so-called Polygamma (Ψ) functions, as

$$A(x,z) = \frac{\mathrm{i}}{\pi} A_0 \left[\Psi\left(\frac{\pi L + 2z - 2\mathrm{i}(x-a)}{2\pi L}\right) - \Psi\left(\frac{\pi L + 2z + 2\mathrm{i}(x-a)}{2\pi L}\right)\right]. \quad (10)$$

Now, the magnetic field components B_x, B_z can be obtained from Eqs. (1) and (8),

$$B_x(x,z) = A_0 L \int_0^{\infty} \frac{\sin[k(x-a)]}{\mathrm{sh}\left(\frac{\pi}{2}kL\right)} \mathrm{e}^{-kz} k dk,$$
$$B_z(x,z) = A_0 L \int_0^{\infty} \frac{\cos[k(x-a)]}{\mathrm{sh}\left(\frac{\pi}{2}kL\right)} \mathrm{e}^{-kz} k dk. \quad (11)$$

Obviously, one can also use Eq. (9) or (10) instead of (8) to derive alternative expressions for B_x and B_z.

At this stage, we consider the limit $L \to 0$ and use Eq. (8) to obtain

$$A(x,z) = A_0 \frac{2}{\pi} \int_0^{\infty} \frac{\sin[k(x-a)]}{k} \mathrm{e}^{-kz} dk = A_0 \frac{2}{\pi} \arctan\frac{x-a}{z}. \qquad (L = 0) \quad (12)$$

This expression shows that magnetic field lines are straight lines described by $x - a = z$. In addition, in the limit $L \to 0$ Eqs. (11) reduce to

$$B_x(x,z) = A_0 \frac{2}{\pi} \int_0^{\infty} \sin[k(x-a)] \mathrm{e}^{-kz} dk = A_0 \frac{2}{\pi} \frac{x-a}{(x-a)^2 + z^2}, \qquad (L = 0)$$

$$B_z(x,z) = A_0 \frac{2}{\pi} \int_0^\infty \cos[k(x-a)] \mathrm{e}^{-kz} dk = A_0 \frac{2}{\pi} \frac{z}{(x-a)^2+z^2}. \qquad (L=0)$$

These are the magnetic field components of an infinitely thin and long line source (compare with Eq. [2.1.2] in Mackay & Priest 1996). Therefore, despite using Eq. (7) to (arbitrarily) define the internal structure of a photospheric source, when its size is reduced to zero the coronal magnetic field becomes that of a line source, as expected.

In addition, it can be of interest to link the scaling parameter L in Eq. (7) with the source width. From Eqs. (1) and (7), the vertical magnetic field component has the following photospheric distribution,

$$B_z(x,0) = \frac{A_0}{L} \cosh^{-2}\left(\frac{x-a}{L}\right).$$

This expression shows that B_z decreases symmetrically about the source centre, $x=a$, from the maximum value A_0/L as $|x-a|$ increases. We now define the width of the photospheric magnetic source as the distance between the points about $x=a$ at which B_z has decreased by a factor e with respect to the central value. It turns out that, with this definition, the width of the photospheric source is about $2L$, which gives a clear physical interpretation of the parameter L.

Fig. 1 shows typical examples of two sources of different extents centred about the origin ($a=0$). Given that the magnetic structure is y-invariant, it is represented by a cut through a $y = const$ plane in this figure and the subsequent ones of the same kind. Fig. 1a, with $L=0.1$ Mm, resembles the magnetic field produced by an infinitely thin line source (Eq. [12]), while the structure shown in Fig. 1b is generated by a broad source of half-width $L=10$ Mm. It is apparent the difference between the two magnetic field configurations not too far from the source.

Having obtained the solution for an individual, unipolar magnetic element, we can immediately generalise the previous results to the case of an arbitrary number of such localised sources or sinks placed on the photosphere. The related flux function $A_s(x,z)$ is then a superposition of flux functions $A_n(x,z)$ of each of the N individual unipolar magnetic regions,

$$A_s(x,z) = \sum_{n=1}^{N} A_n(x,z), \qquad (13)$$

where the A_n are given by Eq. (8),

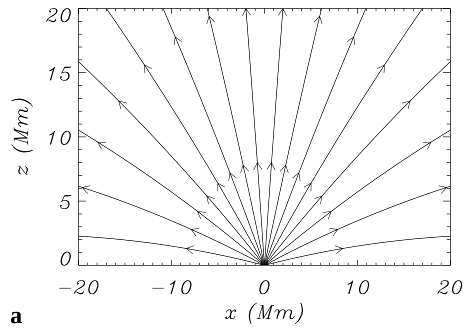
$$A_n(x,z) = A_0^{(n)} L_n \int_0^\infty \frac{\sin[k(x-a_n)]}{\sinh\left(\frac{\pi}{2}kL_n\right)} \mathrm{e}^{-kz} dk,$$

with $A_0^{(n)}$, L_n and a_n the parameters describing the strength, width and position of the n-th magnetic element.

4.2. Uniform background magnetic field

A large scale, uniform, horizontal background field ($\mathbf{B}_{back} = B_{0x}\hat{\mathbf{e}}_x$) has a flux function $A_{back}(x,z)$ clearly given by

$$A_{back} = -B_{0x} z, \quad B_{0x} = \mathrm{const}. \qquad (14)$$



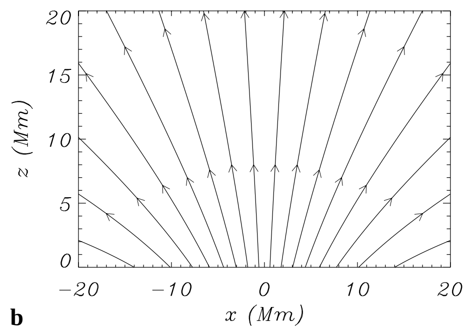


Fig. 1a and b. Examples of the coronal magnetic field pattern from a single source with half-width **a** L=0.1 Mm, and **b** L=10 Mm.

The total flux function of a group of magnetic elements in such a background field is then a superposition of expressions (13) and (14),

$$A(x,z) = A_s(x,z) + A_{back}(x,z), \qquad (15)$$

where, again, the global magnetic field lines are given as contours $A(x,z) = const$.

5. Model for cancelling magnetic features

We first consider a dipolar magnetic configuration produced by a pair of photospheric magnetic elements of equal extent ($L_1 = L_2 \equiv L$), symmetrically placed about the origin ($-a_1 = a_2 \equiv a$) and with opposed flux strength ($-A_0^{(1)} = A_0^{(2)} \equiv A_0$). This means that, for A_0 positive, the magnetic field emerges from the photosphere in the region around $x=+a$ and sinks around $x=-a$. Moreover, $2a$ is simply the distance between the centre of the sources.

A background horizontal magnetic field is added to the dipolar field, so the total flux function is given by (see Eq. [15])

$$A(x,z) = A_0 \left\{ -\frac{B_{0x}}{A_0} z + \frac{\mathrm{i}}{\pi} \left[\Psi\left(\frac{\pi L + 2z - 2\mathrm{i}(x-a)}{2\pi L}\right) - \Psi\left(\frac{\pi L + 2z + 2\mathrm{i}(x-a)}{2\pi L}\right) - \Psi\left(\frac{\pi L + 2z - 2\mathrm{i}(x+a)}{2\pi L}\right) + \Psi\left(\frac{\pi L + 2z + 2\mathrm{i}(x+a)}{2\pi L}\right) \right] \right\}. \qquad (16)$$

The shape of magnetic field lines will then depend on L, a and B_{0x}/A_0, which are related to the linear extent of the photo-

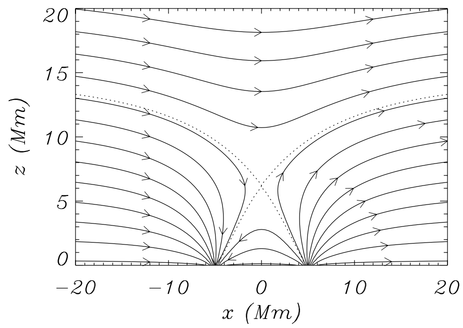


Fig. 2. Coronal magnetic field (solid lines) produced by a dipole with $a = 5$ Mm in a horizontal background field. Other parameters are $L = 0.5$ Mm and $B_{0x}/A_0 = 0.1$ Mm^{-1}. The separatrix (dotted line) marks the boundary between domains containing field lines of different origin. The X-point in the separatrix is a location where magnetic field reconnection can take place.

spheric magnetic elements, $2L$, their separation, $2a$, and the ratio of the background magnetic field to the vertical field at the centre of each of the sources, LB_{0x}/A_0.

This configuration, with infinitely thin photospheric sources, was used by Priest et al. (1994) to model the process by which two photospheric magnetic elements of opposite polarity come together and disappear (cancelling magnetic features). The model starts with the sources widely separated and slowly approaching one another (pre-interaction phase). At this time there is no X-point above the photosphere (see Fig. 4i in Priest et al. 1994) but when the two elements come at a distance

$$2d_0 \equiv 2\frac{4}{\pi}\frac{A_0}{B_{0x}}$$

a null point forms on the photosphere (see their Fig. 4ii). As the sources keep getting closer, this X-point first rises into the corona and then goes down, reaching the photosphere again when the two sources completely annihilate each other ($a = 0$). For source separations, $2a$, between $2d_0$ and zero the magnetic configuration is that in Fig. 4iv of Priest et al. (1994) or our Fig. 2 (interaction phase). During this stage field lines reconnect at the null point and the energy released by this process results in the formation of an X-ray bright point. The interaction phase overlaps with the cancellation phase, during which the sources flux is slowly eroded until only the excess flux contained in the initially strongest element remains.

Because of the symmetry of the dipolar plus background field structure used here, the null point always lies at $x = 0$. To determine its height, $z = c$, one needs the x-component of the magnetic field (cf. Eqs. [1] and [16]),

$$\begin{aligned} B_x(x,z) \;=\; & A_0 \left\{ \frac{B_{0x}}{A_0} - \frac{\mathrm{i}}{\pi^2 L}\left[\Psi'\left(\frac{\pi L + 2z - 2\mathrm{i}(x-a)}{2\pi L}\right)\right.\right. \\ & - \;\Psi'\left(\frac{\pi L + 2z + 2\mathrm{i}(x-a)}{2\pi L}\right) \\ & - \;\Psi'\left(\frac{\pi L + 2z - 2\mathrm{i}(x+a)}{2\pi L}\right) \\ & \left.\left. + \;\Psi'\left(\frac{\pi L + 2z + 2\mathrm{i}(x+a)}{2\pi L}\right)\right]\right\}, \end{aligned}$$

with $\Psi'(u) = \frac{d\Psi(u)}{du}$. The height of the null point follows from the equation $B_x(0,c) = 0$, i.e.

$$\begin{aligned} 0 \;=\; & \frac{B_{0x}}{A_0} - \frac{2\mathrm{i}}{\pi^2 L}\left[\Psi'\left(\frac{\pi L + 2c + 2\mathrm{i}a}{2\pi L}\right)\right. \\ & \left. - \;\Psi'\left(\frac{\pi L + 2c - 2\mathrm{i}a}{2\pi L}\right)\right], \end{aligned} \tag{17}$$

in which B_{0x}/A_0, L and a are free parameters. In the limit of infinitely thin sources this expression reduces to

$$c^2 = \left[\frac{4}{\pi}\frac{A_0}{B_{0x}} - a\right] a \equiv (d_0 - a)a. \qquad (L = 0) \tag{18}$$

We start by computing the positions of the magnetic elements for which the null point is on the photosphere, i.e. for which $c = 0$. Following Priest et al. (1994) we call d the particular value of a for which this happens and plot the results in Figs. 3a and 3b. To describe how to interpret these figures, let us consider Fig. 3a and the particular value $B_{0x}/A_0 = 0.1$ Mm^{-1}. First, for $L = 0$ the two values $d = d_0 \simeq 13$ Mm and $d = 0$, which mark the beginning and end of the interaction phase, are recovered. Moreover, for $0 < L < 8$ Mm the two values of d become increasingly larger with respect to the $L = 0$ case. In other words, the interaction phase begins (and magnetic reconnection starts to take place) when the two sources are at a distance larger than $2d_0$ and finishes well before the total flux in the two photospheric elements completely cancels ($d > 0$). These are the logical consequences of dealing with extense photospheric magnetic sources.

More surprising, however, is the change of behaviour for $L > 8$ Mm. A magnetic null point forms above the photosphere only for a small range of distances between the two elements and for $L > 11$ Mm the null point does not form at all. Therefore, if the two sources are "too large" their magnetic flux cancels without the existence of coronal reconnection. Larger values of the ratio B_{0x}/A_0 lead to similar conclusions, with the formation of the X-point up to a maximum width of photospheric sources, which is inversely proportional to B_{0x}/A_0. Fig. 3b shows the same results using the relative strength of the background field to the photospheric flux as the independent parameter. For infinitely thin sources ($L = 0$) the interaction phase takes place between source positions $d = d_0$ and $d = 0$. For finite sources, however, reconnection occurs only when the background field is not "too strong" or the magnetic elements are not "too large".

Another matter of interest is the change of position of the X-point during the interaction phase. The height of the null point, c, can be obtained from Eq. (17) for extense sources or from Eq. (18) for infinitely thin photospheric sources. The results (see Fig. 4a) stress that coronal reconnection, characterised by $c > 0$, is less important as the ratio of the background magnetic field to the flux of each of the elements increases. Rising the background field intensity has the effect of reducing the range of positions for which the X-point exists and of lowering its height. In addition, Fig. 4b indicates that the null point reaches its maximum height for $L = 0$ and illustrates the fact that, for $L > 0$, it goes down to the photosphere before the flux of the two elements has totally

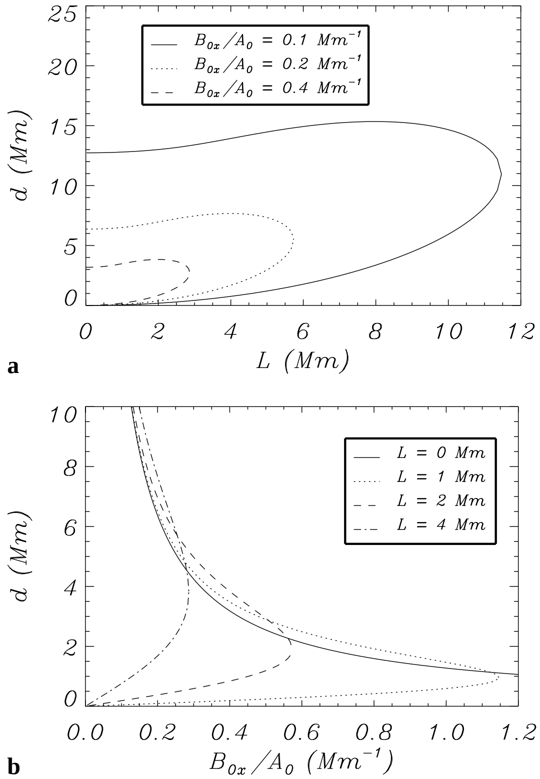


Fig. 3a and b. Position of sources, $d \equiv a$, for which the X-point lies on the photosphere. **a** d versus L taking B_{0x}/A_0 as a parameter. **b** d versus B_{0x}/A_0 taking L as a parameter

cancelled. Also, wide, weak sources (i.e. with large LB_{0x}/A_0 values) may result in flux cancellation without energy liberation in the corona.

The maximum height attained by the X-point, c_{max}, follows from the condition

$$\frac{\partial c}{\partial a} = 0. \tag{19}$$

From Eq. (18), an analytical expression for c_{max} can be obtained for $L = 0$,

$$c_{max} = \frac{2}{\pi}\frac{A_0}{B_{0x}} \equiv \frac{d_0}{2}. \qquad (L = 0)$$

A similar formula cannot be derived for $L \neq 0$, since $c = c(a)$ is known from Eq. (17) in implicit from. One can, however, take the partial derivative with respect to a of this expression and impose condition (19) to obtain

$$\Psi''\left(\frac{\pi L + 2c + 2\mathrm{i}a}{2\pi L}\right) + \Psi''\left(\frac{\pi L + 2c - 2\mathrm{i}a}{2\pi L}\right) = 0. \tag{20}$$

Then, Eqs. (17) and (20) provide c_{max} and the value of a for which this maximum height of the X-point is attained as functions of the parameters L and B_{0x}/A_0. The results, plotted in Fig. 5, reinforce previous conclusions in that larger values of L or B_{0x}/A_0 tend to bring down the null point and to shorten or even remove the interaction phase, in which the X-ray bright point forms.

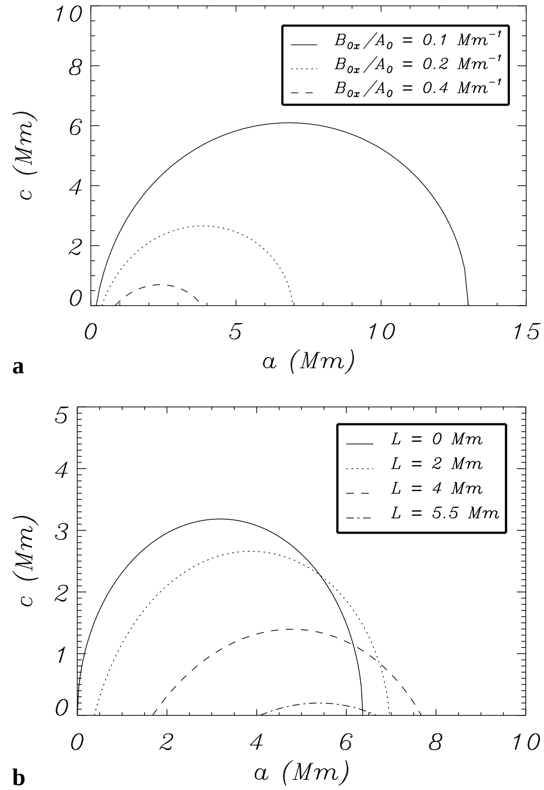


Fig. 4a and b. Height of the X-point versus half the distance between sources for **a** a fixed photospheric element half-width ($L = 2$ Mm) and different values of B_{0x}/A_0; and **b** a fixed ratio of the background magnetic field strength to the dipole strength ($B_{0x}/A_0 = 0.2$ Mm^{-1}) and different values of L.

6. Model for prominence formation

Recently, quadrupolar configurations made of point sources have been used to model qualitatively the formation of solar prominences (Priest et al. 1996). In this section the same problem is addressed using the infinitely long, wide sources introduced before. One starts with a photospheric quadrupole made of two sources of strength $+A_0$ and $-A_0$ placed at $x = -b$ and $x = +b$, respectively, flanking a second pair of sources of strength $-A_0$ and $+A_0$ positioned at $x = -a$ and $x = +a$ ($b > a > 0$), respectively. All four sources have the same half-width, L, and a background horizontal field $\mathbf{B}_{back} = B_{0x}\hat{\mathbf{e}}_x$ is assumed. Next, the two magnetic elements at $x = \pm a$ are allowed to approach one another so that a null point first forms on the photosphere and then raises into the corona (see Fig. 6). During this process the field line that first reconnects on the photosphere moves upwards and drags photospheric mass and magnetic flux to prominence heights. According to Priest et al. (1996), when the two inner sources overlap ($a = 0$), their magnetic flux cancels and the cold plasma captured from the photosphere remains in equilibrium at a typical height of some Mm.

Now, we want to assess the influence of the width of the photospheric sources on the process of flux and mass capture to form a quiescent prominence. Following the procedure outlined

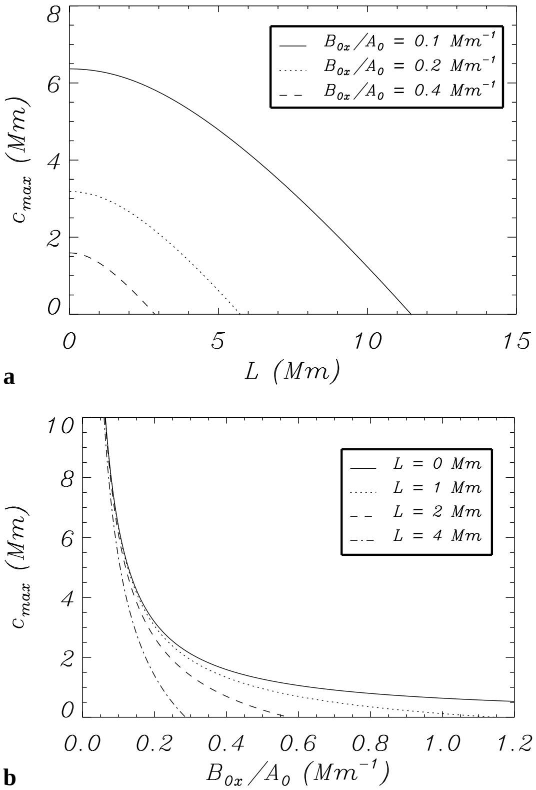


Fig. 5. a Maximum height of the X-point versus L taking B_{0x}/A_0 as a parameter. **b** Maximum height of the X-point versus B_{0x}/A_0 taking L as a parameter.

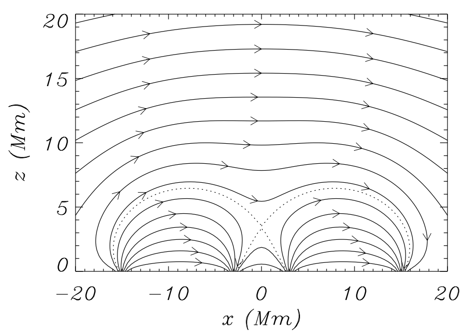


Fig. 6. Coronal magnetic field (solid lines) produced by a quadrupole with $a = 3$ Mm (position of inner sources), $b = 15$ Mm (position of outer sources), $L = 0.5$ Mm and $B_{0x}/A_0 = 0.1$ Mm^{-1}. The separatrix is represented by the dotted line.

in the previous section it is simple to derive the expression for the total flux function,

$$\begin{aligned} A(x,z) \; = \; A_0 \Bigg\{ -\frac{B_{0x}}{A_0} z + \frac{\mathrm{i}}{\pi} \Bigg[& -\Psi\left(\frac{\pi L + 2z - 2\mathrm{i}(x-b)}{2\pi L}\right) \\ & + \Psi\left(\frac{\pi L + 2z + 2\mathrm{i}(x-b)}{2\pi L}\right) \\ & + \Psi\left(\frac{\pi L + 2z - 2\mathrm{i}(x+b)}{2\pi L}\right) \\ & - \Psi\left(\frac{\pi L + 2z + 2\mathrm{i}(x+b)}{2\pi L}\right) \\ & + \Psi\left(\frac{\pi L + 2z - 2\mathrm{i}(x-a)}{2\pi L}\right) \\ & - \Psi\left(\frac{\pi L + 2z + 2\mathrm{i}(x-a)}{2\pi L}\right) \\ & - \Psi\left(\frac{\pi L + 2z - 2\mathrm{i}(x+a)}{2\pi L}\right) \\ & + \Psi\left(\frac{\pi L + 2z + 2\mathrm{i}(x+a)}{2\pi L}\right) \Bigg] \Bigg\}, \end{aligned} \tag{21}$$

and the horizontal magnetic field component,

$$\begin{aligned} B_x(x,z) \; = \; A_0 \Bigg\{ \frac{B_{0x}}{A_0} - \frac{\mathrm{i}}{\pi^2 L} \Bigg[& -\Psi'\left(\frac{\pi L + 2z - 2\mathrm{i}(x-b)}{2\pi L}\right) \\ & + \Psi'\left(\frac{\pi L + 2z + 2\mathrm{i}(x-b)}{2\pi L}\right) \\ & + \Psi'\left(\frac{\pi L + 2z - 2\mathrm{i}(x+b)}{2\pi L}\right) \\ & - \Psi'\left(\frac{\pi L + 2z + 2\mathrm{i}(x+b)}{2\pi L}\right) \\ & + \Psi'\left(\frac{\pi L + 2z - 2\mathrm{i}(x-a)}{2\pi L}\right) \\ & - \Psi'\left(\frac{\pi L + 2z + 2\mathrm{i}(x-a)}{2\pi L}\right) \\ & - \Psi'\left(\frac{\pi L + 2z - 2\mathrm{i}(x+a)}{2\pi L}\right) \\ & + \Psi'\left(\frac{\pi L + 2z + 2\mathrm{i}(x+a)}{2\pi L}\right) \Bigg] \Bigg\}. \end{aligned} \tag{22}$$

Because of the inherent symmetry in this quadrupolar structure, both the centre of the field line that first reconnects and the null point that forms in the corona lie at $x = 0$. The height of the X-point, c, follows after solving the equation $B_x(0, c) = 0$ with $B_x(x, z)$ given by Eq. (22) (see Fig. 7). Just like in the case of cancelling magnetic features, studied before, the results for $L \to 0$ coincide with those obtained using infinitely thin sources and are qualitatively similar to those for point sources (Priest et al. 1996). In addition, as the width of the elements is increased, the X-point first appears for wider separations of the inner sources, it does not rise so much as for $L = 0$ and reaches down the photosphere before the flux cancellation is complete.

On the other hand, the evolution of the height, h, of the line that first reconnects and then rises when the inner sources approach one another can also be studied. The condition to compute h is that the magnetic flux per unit length in the y-direction, ϕ, vanishes up to this height, i.e.

$$\phi = \int_0^h B_x(0, z) dz = 0,$$

which using Eq. (1) reduces to

$$\phi = A(0, 0) - A(0, h) = 0,$$

with $A(x, z)$ given by Eq. (21). Fig. 8 shows that this magnetic field line starts rising when the sources are far apart, achieves its maximum height for $a \neq 0$ and then goes down, reaching

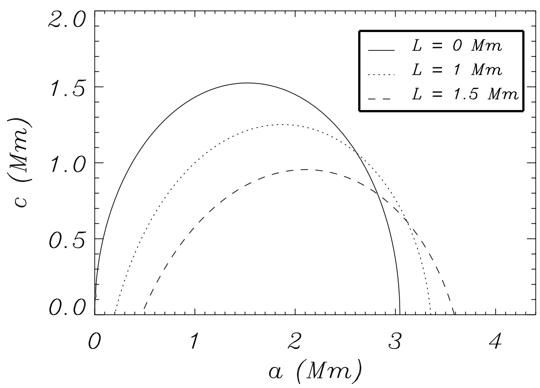


Fig. 7. Height of the X-point versus a for several values of L.

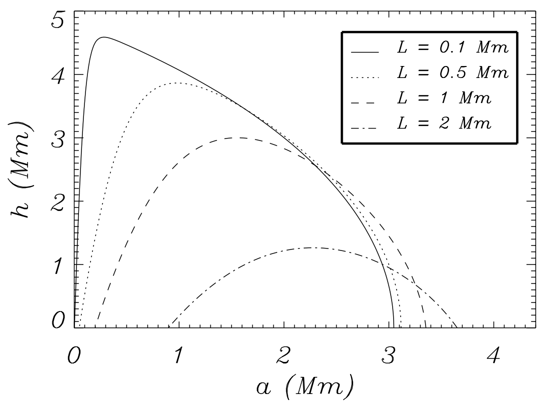


Fig. 8. Height of the first line that reconnects, h, versus a for different values of L.

$z = 0$ before the flux in the two inner sources disappears. In fact, for $L = 2$ the magnetic flux of these two sources has barely started to cancel and already $h = 0$. Therefore, the plasma that rises to form a prominence cannot be maintained above the photosphere unless the flux cancellation process is stopped, because the field line that drags this plasma into the corona inevitably falls down towards the photosphere, as well as the mass supported by it. In the limit of infinitely thin sources the behaviour is similar to the curve for $L = 0.1$ Mm in Fig. 8. As the inner sources approach h continuously increases and takes its largest value as $a \to 0$, but when the sources overlap ($a = 0$) the field line that first reconnects suddenly drops to the photosphere. This behaviour is a consequence of the condition $\phi = 0$, since when the inner sources cancel the coronal X-point disappears and the flux above the photosphere at $x = 0$ has only one sign, so that $\phi = 0$ can only be satisfied for $h = 0$. Consequently, it turns out that full flux cancellation with $L = 0$ destroys the formed prominence, so its existence should be based on a continuous emergence and cancellation of magnetic flux, in order to maintain the supply of mass and flux. It must also be mentioned that the finite width of the sources represents another problem in this model for prominence formation. For example, if the two inner fragments are $2L = 4$ Mm wide, the cold material rises at a maximum height which is about 20% that attained for $L = 0$ (≈ 1 Mm compared to ≈ 4.5 Mm, see Fig. 8). On the other hand, the basic trends found here agree with the results obtained by Démoulin & Priest (1993) for an inverse polarity prominence in a quadrupolar region. In such model, they already showed that to obtain low height dips one needs a corridor nearly free of magnetic field, i.e. large a and L not too big.

7. Discussion and conclusions

In this paper we have described an analytical method to model the coronal magnetic field arising from wide, infinitely long photospheric sources. The model is stationary, but can be used to investigate the slow evolution of a system taken as a series of equilibrium states. The present approach represents an improvement over previous works in that photospheric magnetic elements have a finite width in comparison with the infinitely thin line sources used before.

We have prescribed a particular spatial distribution of magnetic flux in the photospheric fragments. It has first been shown that, as the width of the elements tends to zero the coronal configurations produced by line sources are recovered, both for a single source and for more complicated combination of photospheric magnetic sources and sinks.

The model of cancelling magnetic features proposed by Priest et al. (1994) has been re-examined using extense magnetic sources. It has been found that the width of photospheric elements must be taken into account as it has a strong influence on the formation of a coronal null point and on the subsequent magnetic reconnection during the slow evolution of the system. Thus, wide and/or weak sources are not sufficiently strong as to overpower the background field at low heights and so the energy release resulting in the formation of the coronal X-ray bright point is not produced. During the flux cancellation of strong and/or narrow sources the null point forms in the corona, although its maximum height is always smaller than that for line sources of the same strength. Moreover, the interaction phase (during which the coronal X-point is present) is longest for thin sources and tends to shorten, or even disappear, for wider magnetic fragments.

Concerning the quadrupolar configuration used to model prominence formation, two main conclusions can be extracted: Firstly, as the inner sources approach one another, the neutral point first rises and later returns to the photosphere, with the maximum height attained depending on L as well as on the ratio B_{0x}/A_0. Again, the X-point returns to $z = 0$ for $a \neq 0$, contrary to what happens for point or line sources. Secondly, the first field line that reconnects, assumed to be responsible for dragging photospheric material up to prominence heights, does not remain above the photosphere if the two inner sources are assumed to cancel. Even for $L = 0$ total flux cancellation produces the fall of this field line because of the imposed condition $\phi = 0$. This means that the raised mass cannot be supported high in the corona and that the prominence disappears when the cancellation occurs ($L = 0$) or before it is finished ($L \neq 0$). For this model to be operative, prominences, lasting long within the solar corona, need to be fed by a constant supply of mass and flux provided by a continuous emergence and cancellation of magnetic flux.

Finally, it may also be possible to consider different forms of the photospheric flux distribution, $A(x, 0)$, rather than the one used here (Eq. [7]). This would allow to determine the influence of the flux distribution within the source, although it should not be too important as long as the flux is more or less concentrated towards the source centre.

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