Astron. Astrophys. 351, 883–892 (1999)

ASTRONOMY
AND
ASTROPHYSICS

Investigation of subclustering in 18 rich clusters of galaxies using wavelet analysis

J. Krywult[1], **H.T. MacGillivray**[2], **and P. Flin**[1]

[1] Pedagogical University, Institute of Physics, ul. M.Konopnickiej 15, 25-409 Kielce, Poland
[2] Royal Observatory Edinburgh, Blackford Hill, Edinburgh EH9 3HJ, UK

Received 30 May 1999 / Accepted 1 September 1999

Abstract. The method based on the wavelet transform has been applied to positions of galaxies in 18 rich Abell clusters in a search for the presence of subclustering. This study has shown that substructures are present in about 50% of cases. If our result is applicable generally to clusters (as other studies also indicate), then we conclude that subclustering is an important feature of cluster structure and hence of evolution.

Key words: galaxies: clusters: general

1. Introduction

It has long been known that the distribution of galaxies on the sky shows a high degree of clumping. Most galaxies belong to groups ranging in richness from numerous poor associations up to rich clusters of galaxies such as the Coma and Virgo clusters. Recent studies have furthermore concluded that many galaxy clusters are not spherically symmetric systems in equilibrium. The presence or absence of "substructures" in clusters of galaxies is thus very important for the study of cluster evolution and formation.

The frequency of substructures in clusters has been used in the past to evaluate the density parameter Ω_o (Richstone et al. 1992, Kauffmann & White 1993). By constructing the radial number density profiles for about 100 clusters Baier (1979, 1983) and Baier & Mai (1977, 1978) concluded that as many as 80% of clusters he studied had subclustering. Based on the projected distribution of galaxies, using surface density contour maps for 65 rich galaxy clusters, Geller & Beers (1982) found that substructures are statistically significant in 40% of these cases. The same percentage of subclustering was reported by Dressler & Shectman (1988) from 3-D data (radial velocities of galaxies). West et al. (1988) advocated a lower percentage of subclustering, finding little significant subclustering in the sample studied. Using both projected distributions and radial velocities of galaxies, West & Bothun (1990) found structures in the outer parts of clusters. Bird (1993), using a number of statistical tests on 33 clusters, showed that depending on the method applied from 10% to 40% of them have statistically significant substructures. Moreover, Escalera et al. (1994) applied the wavelet analysis to 16 clusters classified by them as unimodal or bimodal. More recent studies have been carried out by Kriessler & Beers (1997) using an adaptive kernel technique for 56 clusters. They concluded that 57% of these clusters show substructures.

2. Observational data

The data for our analysis come from automatic scans of 48-inch Schmidt telescope plates, performed at the Royal Observatory, Edinburgh using the COSMOS machine. We use the data from the *COSMOS/UKST Southern Sky Object Catalogue* (Yentis et al. 1991). For each galaxy the catalogue gives: magnitude, ellipticity, the galaxy major axis diameter, its position angle and the number of pixels detected within the image above an isophotal threshold set for image detection. The catalogue is 90% complete down to Bj=20^m. Basic parameters of the clusters included in the present study are given in Table 1. The first column contains the cluster name, the second the cluster type after Abell et al. (1989). Columns 3 and 4 give the right ascension and declination (2000) of the cluster centre respectively, Column 5 is the redshift of the cluster from Struble & Rood (1987). Column 6 contains the total number N of galaxies in the cluster region. Obviously, the number of galaxies in the cluster is smaller than the total number of galaxies N in the cluster region. In the analysis we took into account all galaxies in the region of the cluster with magnitude in the range m_3 to $m_3 + 3$, where m_3 is the magnitude of the third brightest galaxy. The typical redshift of the cluster sample is $z = 0.057$.

3. Wavelet analysis

The detection of structure in the regions studied was made by means of the wavelet analysis (Escalera et al. 1994). The wavelet technique is a convolution on a grid of $N \times N$ pixels between the signal $s(r)$ (in our case the angular positions of galaxies) and an analyzing wavelet function $g(r, a)$. In this work, following Escalera & Mazure (1992), we use the two-dimensional radial function called the Mexican Hat given by the formula:

$$g(r,a) = \left(2 - \frac{r^2}{a^2}\right) \exp\left(-\frac{r^2}{a^2}\right) \qquad (1)$$

Send offprint requests to: J. Krywult

Table 1. Observational data

Cluster	Type	α	δ	z	N
A14	C	00 15.2	-23 53	0.0640	629
A85	cD	00 41.6	-09 20	0.0518	643
A119	C	00 56.4	-01 15	0.0440	200
A133	cD	01 02.6	-21 47	0.0604	470
A151	cD	01 08.9	-15 25	0.0526	226
A194	L	01 25.6	-01 30	0.0178	159
A261	cD	01 51.4	-02 14	0.0467	519
A279	cD	01 56.4	$+01$ 03	0.0797	515
A358	C	02 30.2	-13 11	0.0576	224
A496	cD	04 33.6	-13 14	0.0320	604
A644	cD	08 17.4	-07 35	0.0704	1376
A754	cD	09 08.8	-09 38	0.0528	329
A978	F	10 20.5	-06 31	0.0527	518
A1139	I	10 58.1	$+01$ 29	0.0383	239
A1650	cD	12 58.8	-01 45	0.0845	738
A1651	cD	12 59.1	-04 11	0.0825	1140
A1837	cD	14 01.8	-11 09	0.0376	734
A2670	cD	23 54.2	-10 24	0.0745	836

Table 2. Results

Cluster	Struct.	a–Scale	N$_{gal}$	X_{sc}	Y_{sc}	Morph.
A14	A	64	95	124	138	U
A85	A	64	119	109	117	U
A119	A	64	64	129	139	U
A133	A	64	81	121	135	B
	A1	45	55	111	138	
	A2	45	46	213	039	
	B1	16	18	127	121	
	B2	16	16	095	155	
A151	A	64	48	111	118	S
	B	32	23	122	127	
	C	32	17	125	026	
A194	A	64	49	117	119	U
A261	A	64	45	133	187	U
A279	A	64	69	134	127	U
A358	A	64	50	117	118	S
	B	11	8	098	106	
A496	A	64	178	146	141	S
	B	45	115	131	124	
	C	45	84	201	221	
A644	A	64	166	136	116	U
A754	A	64	102	108	123	S
	B	16	19	123	130	
	C	16	17	084	114	
	D	11	6	161	169	
A978	A	64	126	133	115	S
	B1	22	30	127	086	
	B2	22	28	131	150	
A1139	A	64	51	127	121	U
A1650	A	64	119	121	135	S
	B	16	10	178	084	
	C	16	10	097	035	
A1651	A	64	168	134	118	S
	B	32	38	215	032	
A1837	A	64	456	121	184	S
	B	22	90	135	189	
	C	22	53	228	068	
	D	16	55	134	189	
	E	16	44	129	130	
A2670	A	64	181	134	140	U

where r is the distance between the position of a galaxy and a point (x, y) where the wavelet coefficient is calculated, and a is a scalelength for the wavelet in order to form the corresponding set of wavelet coefficients. As a result of the convolution, the signal is transformed into a set of the wavelet coefficients which are given by:

$$w(r, a) = g(r, a) \otimes s(r) \quad (2)$$

Each pixel in the grid has then a corresponding wavelet coefficient associated with it. Using a set of different scales, a, a structure is detected only when its characteristic size is of the order of the applied scale. Following Daubechies (1990), the factor $\sqrt{2}$ from one scale to another ensures, in the case of the Mexican Hat, correct sampling. The field when analyzed with the largest scale will produce a wavelet image showing a single central structure. If the scale decreases, the central structure either remains unchanged or splits into substructures. In this way we can detect all structures present in the map, irrespective of their location or size.

For the analysis presented here, the discrete wavelet was computed on a grid of 256×256 pixels for seven scales increasing from $a = 8$ to 64 (in pixel units), namely $8, 11, 16, 22, 32, 45, 64$ respectively. The corresponding distances in kpc are: 125, 172, 250, 344, 500, 703, 1000 (assuming H$_o$=75 km/(s·Mpc). In order to avoid any edge effects, areas larger than the cluster itself were analyzed. For determining the center of the clusters we used two-dimensional number-density contour maps.

We have modelled the significance of the substructuring detected using Monte Carlo simulations. For each cluster and each scale a, the wavelet analysis was carried out on a set of 1000 spherically symmetric distributions of galaxies containing the same number of points as in the true fields. In order to match the real clusters as closely as possible, the spherically symmetric galaxy distributions were obtained using the real clusters by making the angular coordinates of the galaxies follow a random distribution but leaving the radial coordinates of the galaxies untouched. In this way, we can remove the presence of local substructures, but we have the same form for the radial profiles as for the real clusters.

We assume that a substructure is real if the corresponding wavelet coefficient associated with it is greater than the maximum of coefficients for the modelled distributions. This means that there is a probability of less than 0.001 that the detected (sub)structure is due to random fluctuations. Furthermore, for each scale a only substructures with more than 4 galaxy members in a circle of radius a are noted.

In this way, we detected all structure in the field of the galaxy clusters studied here, and moreover following the work of Escalera et al. (1994) we have undertaken morphological classifi-

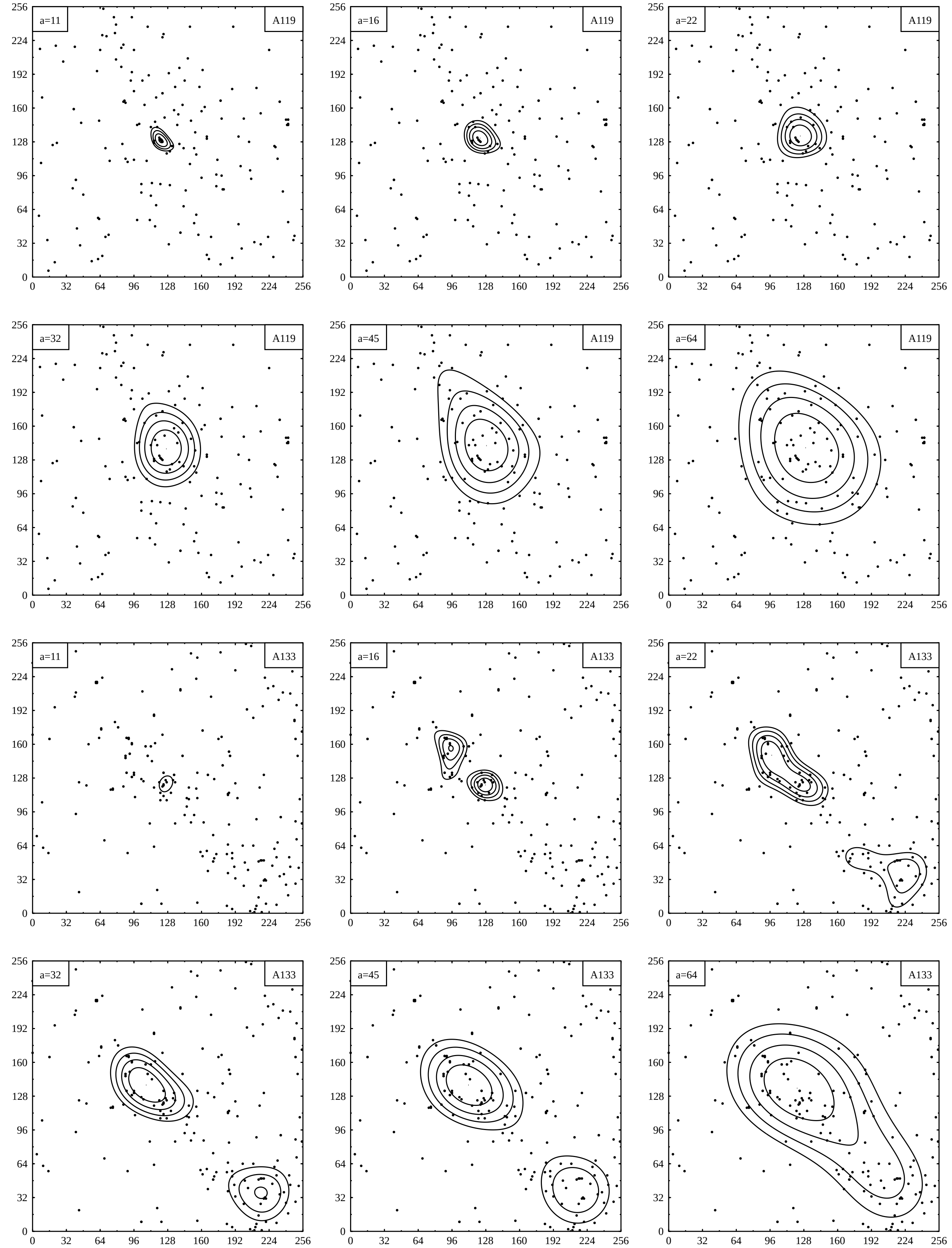

Fig. 1. The distribution of galaxies in A119 and A133 with wavelet images for scales a=11, 16, 22, 32, 45, 64 pixels.

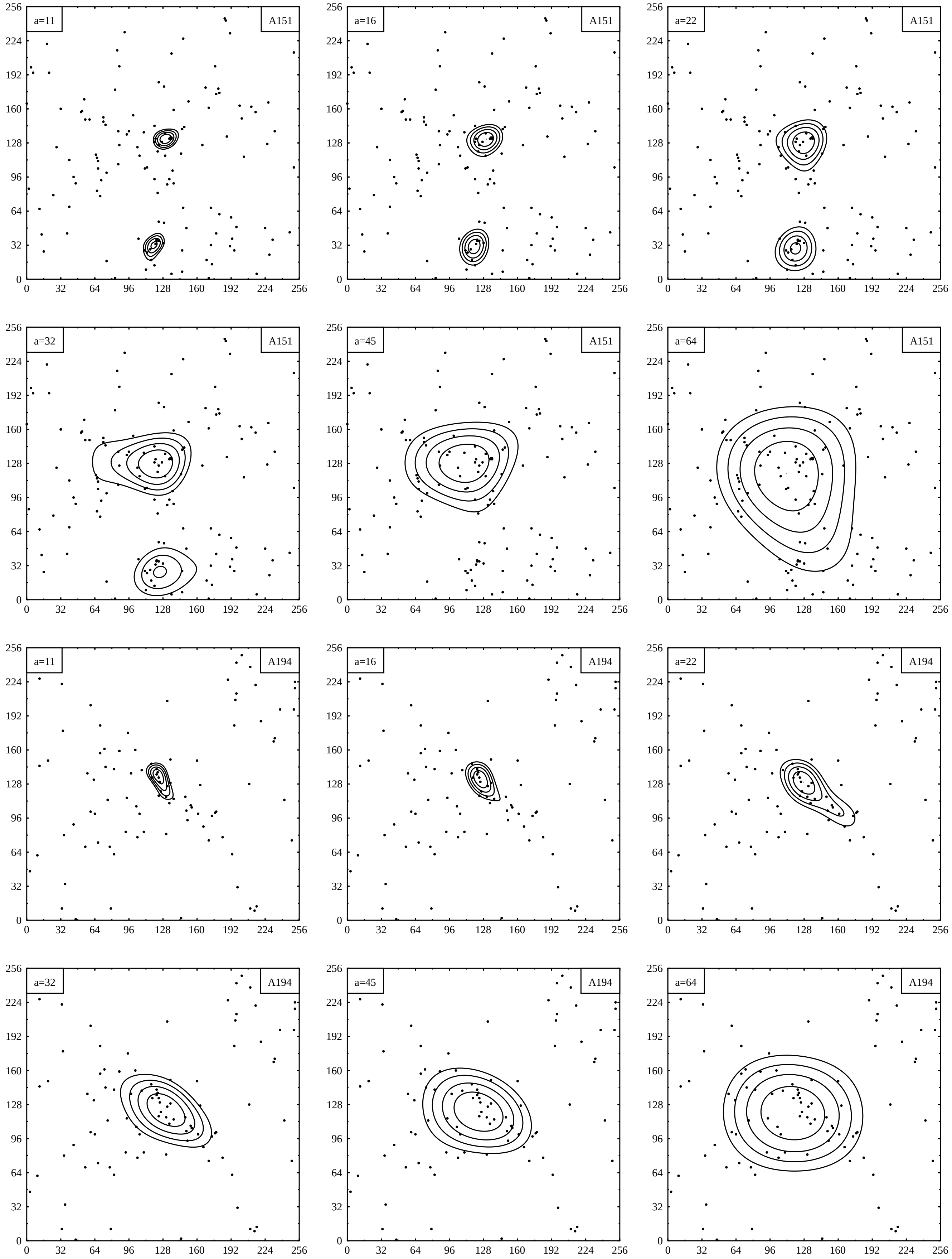

Fig. 2. The distribution of galaxies in A151 and A194 with wavelet images for scales a=11, 16, 22, 32, 45, 64 pixels.

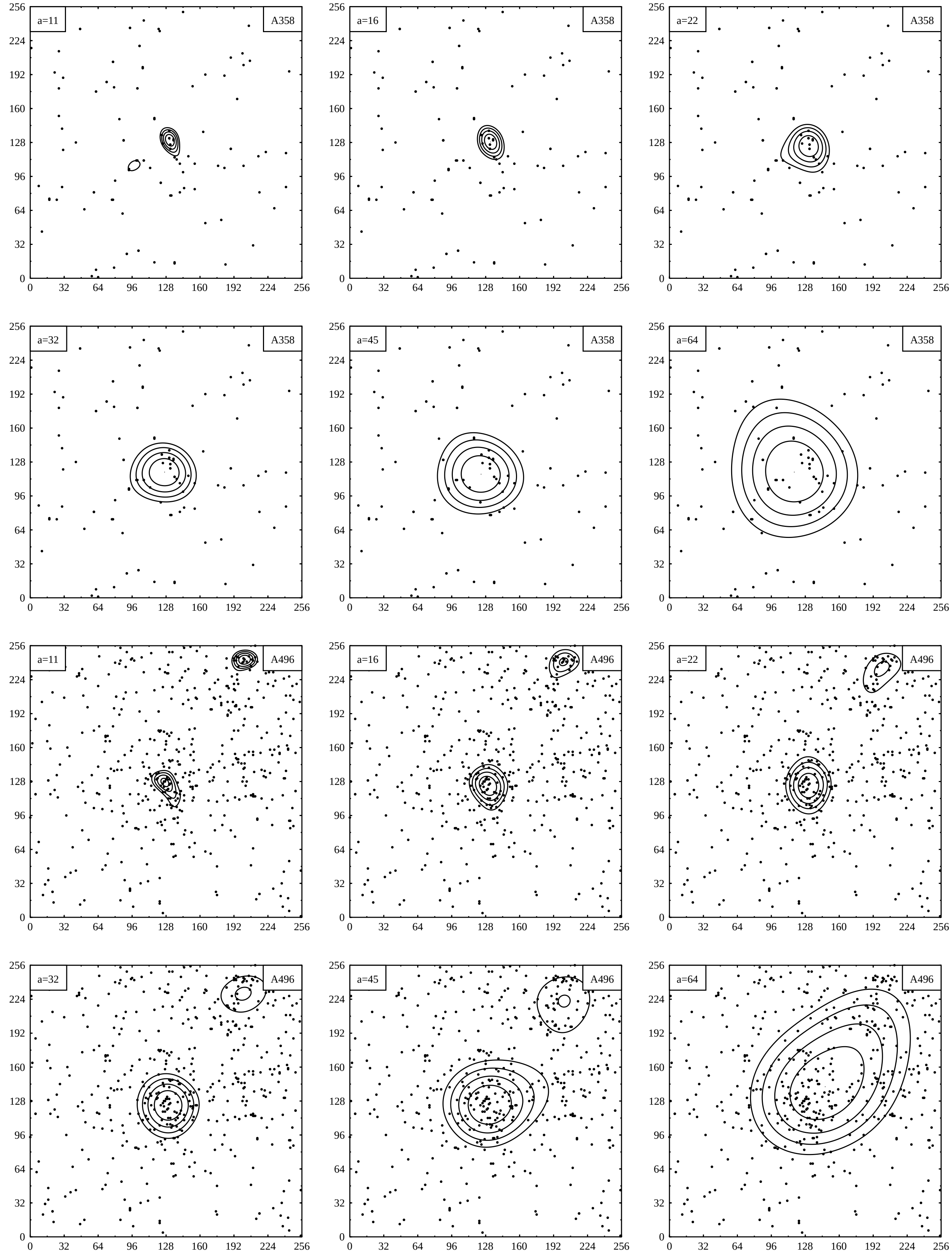

Fig. 3. The distribution of galaxies in A358 and A496 with wavelet images for scales a=11, 16, 22, 32, 45, 64 pixels.

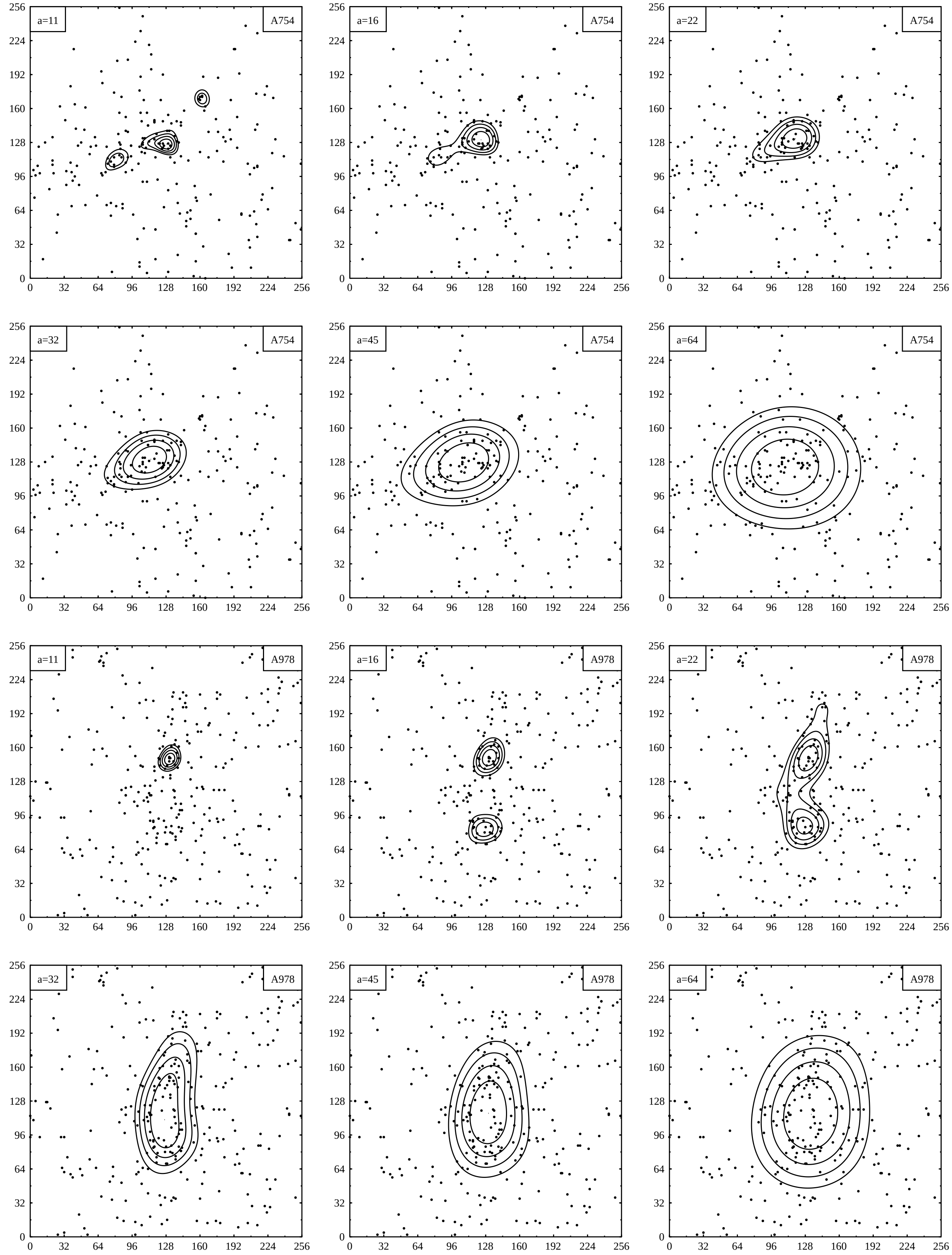

Fig. 4. The distribution of galaxies in A754 and A978 with wavelet images for scales a=11, 16, 22, 32, 45, 64 pixels.

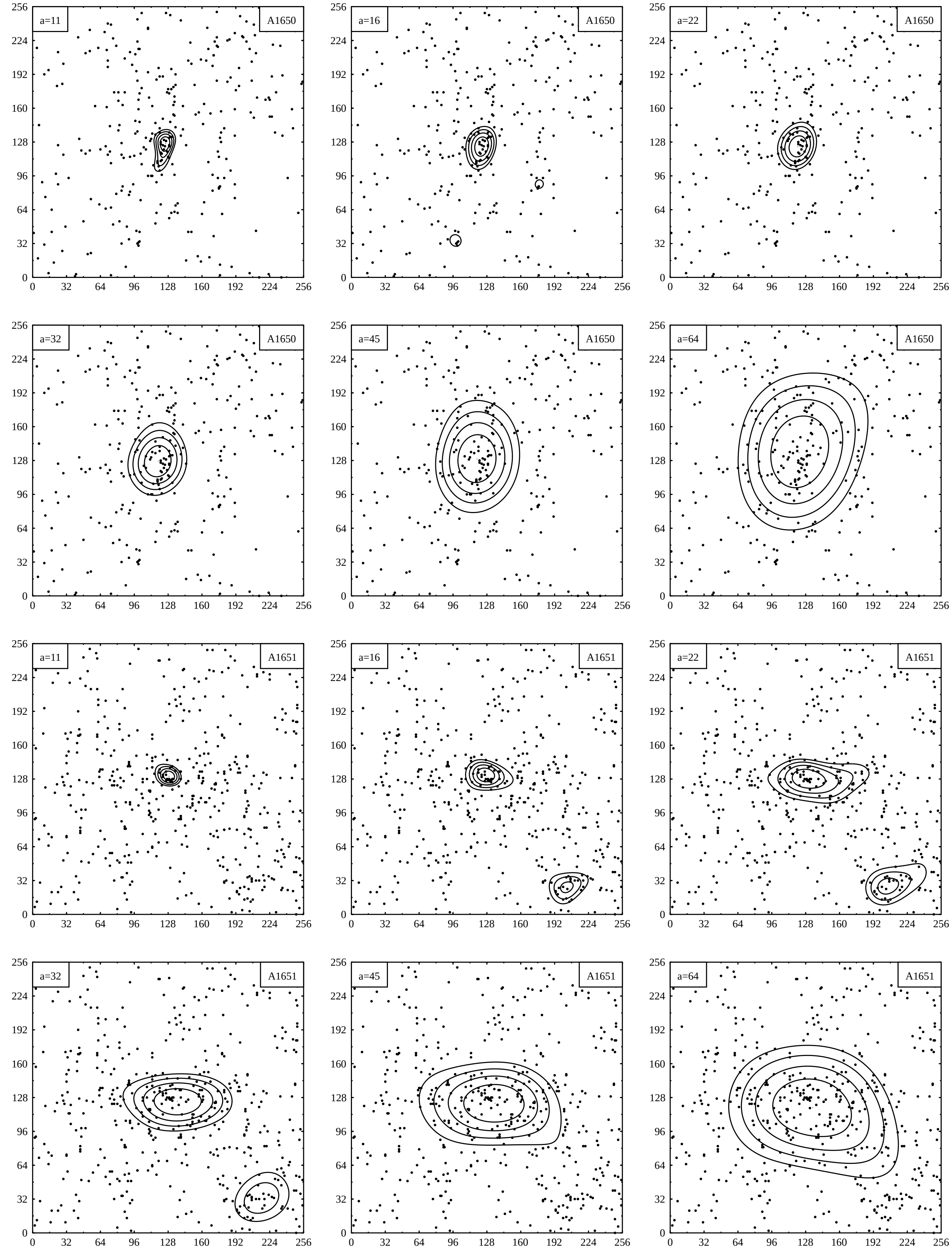

Fig. 5. The distribution of galaxies in A1650 and A1651 with wavelet images for scales a=11, 16, 22, 32, 45, 64 pixels.

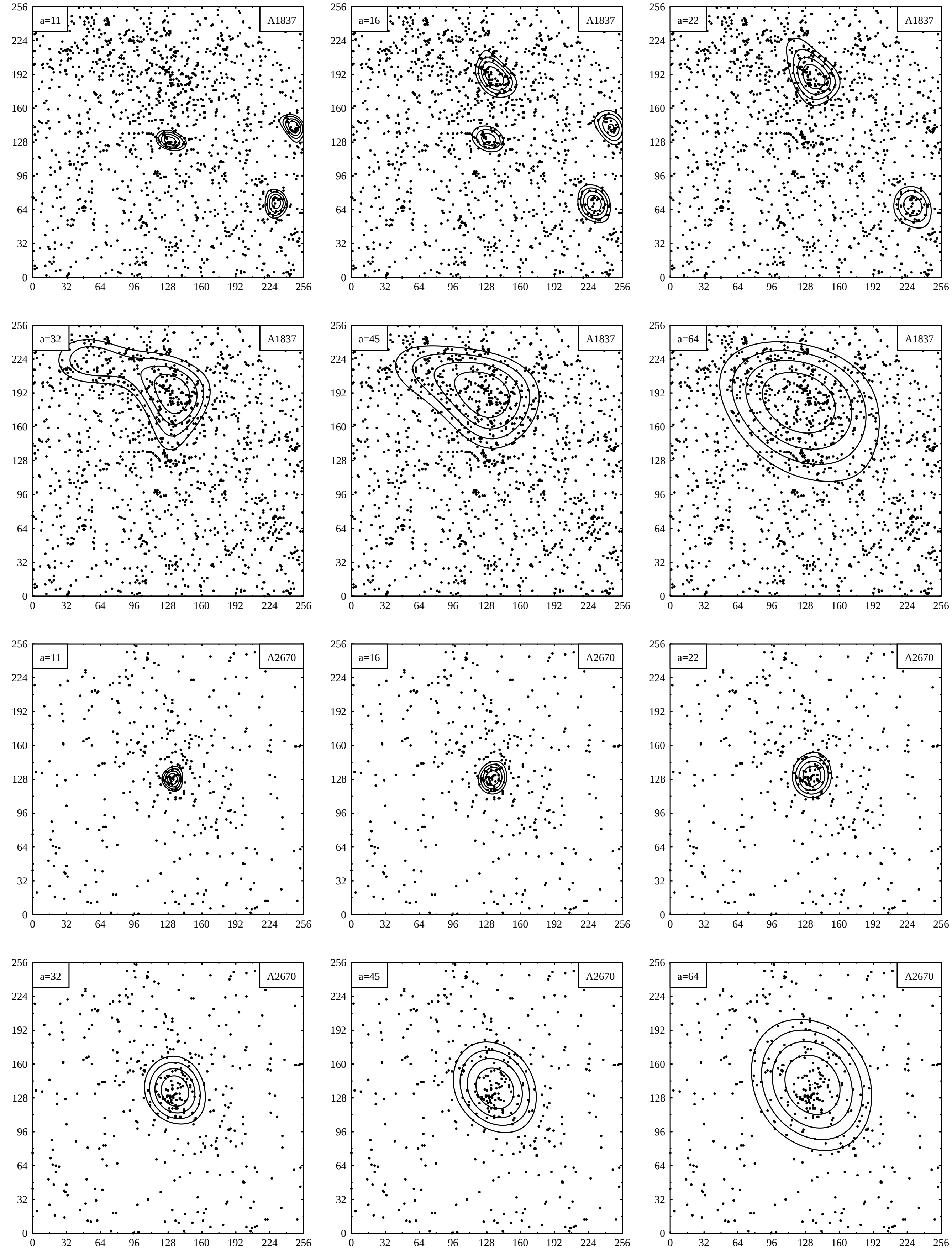

Fig. 6. The distribution of galaxies in A1837 and A2670 with wavelet images for scales a=11, 16, 22, 32, 45, 64 pixels.

Table 3. Substructures in investigated clusters of galaxies

Cluster	Papers[a]												
	1	2	3	4		5		6	7	8	9	10	
	2-D	2-D	3-D	2-D	3-D	2-D	3-D	2-D	2-D	2-D	2-D	2-D	3-D
A14		–						–	–	–	–	–	–
A85										–	–		
A119		+						–	+	–	–	+	–
A133										+	+		
A151	+	+				+	+	+	+	+	+		
A194		+						–	+	–	–	–	–
A261										–	–		
A279										–	–		
A358										–	–		
A496		+						–	+	+	+	–	–
A644										–	–		
A754	+	+	–	+	+	+	+	+	+	+	+		
A978	+	+						–		+	–	+	–
A1139	–	–						–	–	–	–		
A1650										–	–		
A1651										–	–		
A1837										+	+		
A2670	–					+	+	–		–	–	–	+

[a] The consecuitve papers are:
1. Baier (1979, 1983), Baier & Mai (1977, 1978); 2. Bird (1993); 3. Dressler & Shectman (1988); 4. Escalera & Mazure (1992); 5. Escalera et al. (1994); 6. Geller & Beers (1982); 7. Kriessler & Beers (1997); 8. Krywult et al. (1996); 9. Krywult (1997); 10. West & Bothun (1990).

cation of the structures according to three categories: unimodal, bimodal and complex.

The results of our wavelet analysis on the 18 Abell clusters of galaxies are given in Table 2. Column 1 lists the cluster name, Column 2 identifies the structure (M – main cluster; A – dominant central structure; A1, A2 – two components in bimodal cases; B, C – substructures), Column 3 lists the wavelet scales a at which the structures were detected, with values in pixel units for a map of 256×256 pixels, Column 4 gives the number of galaxies belonging to the relevant structures, Columns 5 and 6 give coordinates for the centre of the detected structure, and lastly Column 7 gives the morphology of the clusters (U – unimodal, B – bimodal, S – substructures).

In order to present our results in a reasonably compact manner, from the 18 clusters studied we show in Figs. 1 – 6 only those clusters with detected substructures, together with three clusters without substructuring. The diagrams show the distribution of galaxies and the wavelet images on a grid of 256×256 pixels for six wavelet scales $a = 11, 22, 32, 45, 32, 64$ (in pixel units). It is possible from these to identify unimodal or bimodal clusters and eventually substructures. In general the central A – structure is still seen at small scales, although the corresponding physical structure is only clearly defined on the largest scales.

4. Results

By applying the wavelet analysis to positional data for galaxies in eighteen Abell clusters, we find that 50% of the clusters investigated, viz. A14, A85, A119, A194, A261, A279, A644, A1139, A2670 are unimodal, i.e. show no evidence of any substructures. Significant substructuring is found, however, in six clusters: A133, A151, A496, A978, A1651, and A1837. Moreover, in three clusters: A358, A754 and A1650, the wavelet analysis has detected small structures in the cores or in the fields of clusters only at small wavelet scales a. Thus, the wavelet analysis shows statistically significant substructures in 50% of the clusters in our sample.

In previous papers, the positional data of galaxies from the same sample of clusters was examined by three different methods. Applying the surface number density contour plots, Krywult et al. (1996) found significant substructuring in the clusters A133, A151, A496, A754, A978 and A1837. Moreover, using the symmetry and separation tests proposed by West et al. (1988) substructures were detected (Krywult 1997) in A133, A151, A496, A754 and A1837.

In general, the results presented in this paper are consistent with those of other authors. Using 2-D and 3-D data substructures were reported in A151 and A754 by Geller & Beers (1982), Bird (1993), Escalera et al. (1994), Kriessler & Beers (1997). In the case of A978, evidence of substructuring was found by West & Bothun (1990) and by Bird (1993), while Geller & Beers (1982) reported no evidence for substructures. However, for some of the above-mentioned clusters our results are different. From three-dimensional data, West & Bothun (1990) pointed out a strong signal for subclustering in A2670, while in the case of A496 no evidence for significant substructure has been noted.

5. Conclusions

The method based on the wavelet transform has been applied to detect substructures in the projected distributions of galaxies in 18 rich Abell clusters. The significance of substructuring detected has been calibrated using simulated distributions by means of Monte Carlo modelling. On applying the wavelet analysis to projected distributions of galaxies in our sample of clusters, we conclude that significant structure is present in 9 of the 18 (i.e. 50%) of cases, viz. A133, A151, A358, A496, A754, A978, A1650, A1651 and A1837.

Our results indicate that a large fraction of clusters have substructures in their projected galaxy distributions. The comparison between the presence of substructures in our 18 studied clusters and those reported by other authors is presented in Table 3. The sign "–" denotes non-detection of subclustering, whereas "+" indicates detection. "Blank" indicates that the cluster was not included in our sample of clusters studied. Comparison of our and other recent studies indicates a correlation between the fraction of substructures detected in 2-D and 3-D data. Thus, the analysis of substructures in clusters of galaxies based only on 2-D data can reveal a statistically correct percentage of subclustering, disregarding some differences for individual objects (Politzer & Preskill 1986). We suggest that in order to conduct as full an analysis as possible, several different methods should be applied to data in order to allow a better structural analysis. Each method is sensitive to different features of subclustering, as well as allowing the detection of structures at a variety of different scales.

We plan to apply different statistical methods of substructure analysis in galaxy clusters to a much larger sample of Abell clusters, thus allowing us to determine more accurately the fraction of galaxy clusters with significant substructuring.

Acknowledgements. This work was partialy suported by grants KBN/IBMSP/WSP/Kielce/073/1998 and KBN/WSP/Kielce/BW236 /98.

References

Abell G.O., Corwin H.G. Jr., Olowin R.P., 1989, A&AS 70, 1

Baier F.W., 1979, Astron. Nachr. 300, 133

Baier F.W., 1983, Astron. Nachr. 304, 211

Baier F.W., Mai W., 1977, Astron. Nachr. 298, 301

Baier F.W., Mai W., 1978, Astron. Nachr. 299, 69

Bird C.M., 1993, Ph.D. Thesis, University of Minnesota

Daubechies I., 1990, I.E.E.E. Trans. on Information Theory 36, 961

Dressler A., Shectman S.A., 1988, AJ 95, 985

Escalera E., Biviano A., Girardi M., et al., 1994, AJ 423, 539

Escalera E., Mazure A., 1992, ApJ 388, 23

Geller M.J., Beers T.C., 1982, PASP 94, 421

Kauffmann G., White S.D.M., 1993, MNRAS 261, 921

Kriessler J.R., Beers T.C., 1997, AJ 113, 80

Krywult K., 1997, Kinematics and Physics of Celestial Bodies 5, 36

Krywult J., MacGillivray H.T., Flin P., 1996, Proc. of the 27-th Meeting of the Polish Astronomical Soc., p. 92

Politzer H.D., Preskill J.P., 1986, Phys. Rev. Letters 56, 99

Richstone D., Loeb A., Turner E.L., 1992, ApJ 393, 477

Struble M.F., Rood H.J., 1987, ApJS 63, 555

West M.J., Bothun G.D. 1990, ApJ 350, 36

West M.J., Oemler A., Dekel A., 1988, ApJ 327, 1

Yentis D.J., Cruddace R.G., Gursky H., et al., 1991, In: MacGillivray H.T., Thomson E. (eds.) Digitised Optical Sky Surveys. p. 67