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ASTRONOMY AND ASTROPHYSICS

On the possible existence of a self-regulating hydrodynamical process in slowly rotating stars

I. Setting the stage

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Abstract. It has been known for a long time (Mestel 1953) that the meridional circulation velocity in stars, in the presence of μ-gradients, is the sum of two terms, one due to the classical thermal imbalance (Ω-currents) and the other one due to the induced horizontal μ-gradients (μ-induced currents, or μ-currents in short). In the most general cases, μ-currents are opposite to Ω-currents. Simple expressions for these currents are derived under some simplifying physical assumptions presented in the text, and their physical interpretations are discussed. Computations of the Ω and μ-currents in a 0.8 $M_\odot$ halo stellar model including classical element settling show that the μ-currents are larger (in absolute values) than the Ω-currents in all the star: some new physics has thus to be invoked in this case.

We show here how such processes could possibly lead to a quasi-equilibrium stage in which both the circulation and the helium settling could be cancelled out. As lithium diffuses in the same way as helium, we expect a very small lithium concentration gradient below the convective zone in "plateau stars" (main-sequence Pop II stars), much smaller than the one expected for pure element settling. This could possibly account for the very small dispersion observed for the lithium abundances at the surface of these stars. This should also have important consequences in other contexts which will be discussed in forthcoming papers.

The present computations show that element settling in slowly rotating stars leads to surface abundances which depend on the competition between μ-currents and Ω-currents, in a way which had not been taken into account in previous computations. This may change our general understanding of the diffusion processes of chemical species in rotating stars.

Key words: diffusion – hydrodynamics – stars: abundances – stars: Population II

1. Introduction

The present study was motivated by the so-called "lithium plateau paradox" (Vauclair 1999), which may be formulated in the following way:

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- From spectroscopic observations, the lithium abundance in main sequence Pop II field stars with effective temperatures larger than 5500 K is remarkably constant. Since the first observations of the lithium plateau by Spite & Spite (1982), many abundance determination have confirmed the constancy of the lithium abundance in most of these stars, although a small slope may exist as a function of effective temperature (Thorburn 1994, Ryan et al. 1996, Spite et al. 1996, Bonifacio & Molaro 1997). Moreover the dispersion around the average value is extremely small. Whether this dispersion is real or below the observational errors is still a subject of discussion. (Deliyannis et al. 1993, Bonifacio & Molaro 1997, Molaro 1999). In globular clusters however, some dispersion exists for subgiants close to the turn-off (Deliyannis et al. 1995, Boesgaard et al. 1998).
- On the other hand, theoretical computations lead to predictions of large variations of the lithium abundance from star to star, unless some "ad hoc" process forces the lithium abundance to remain constant (Charbonnel & Vauclair 1998, and references therein). The reason for these predicted variations is related to the diffusion processes which take place in the radiative regions inside the stars, below the outer convective zones. Due to pressure and temperature gradients, lithium settles down towards the stellar center, as well as helium and other heavier elements. Mixing induced by rotation, internal waves, or mass loss related motions may slow down the settling process, but then it brings up to the convective zone matter in which lithium has been destroyed by nuclear reactions.

These lithium depleting processes have been extensively studied in the litterature: (e.g. Michaud et al. 1984, Vauclair 1988, Pinsonneault et al. 1992, Vauclair & Charbonnel 1995 and 1998). The results are well known and can be summarized as follows:

- If the radiative zones were completely stable, lithium would be depleted in Pop II stars due to settling, and the effect would increase with effective temperature. As a consequence, the lithium abundances should decrease with increasing $T_{\rm eff}$ in contradiction with the observations. This result suggests that some macroscopic motion must occur and compete with the lithium settling. (e.g. Michaud et al. 1984, Vauclair 1988, Chaboyer et al. 1992).

- Rotation-induced mixing, as prescribed by Zahn (1992), or Pinsonneault et al. (1992), could account for the observations. However the small dispersion of the observed abundances is difficult to interpret in this framework (Vauclair and Charbonnel 95) and gives in any case an upper limit on the initial lithium value (Pinsonneault et al. 1998).
- Mass-loss could also be a possible explanation (Vauclair & Charbonnel 1995). However all the stars should have suffer exactly the same mass loss rate, about 30 times the solar wind rate, during all their lives.

In all cases, the lithium plateau can be accounted for only with “ad hoc” hypothesis, where some parameter is assumed constant (or with very small variations) in all stars (mass loss rate, rotation rate...). This is not satisfying, and observers often prefer to forget about this physics and decide that the lithium abundance observed in Pop II stars must be the primordial one.

This interpretation however is in contradiction with the strong improvements of stellar physics obtained these last few years. The theory of stellar structure and evolution includes the best available plasma physics, with new equations of states, opacities, nuclear reaction rates and element settling. Helioseismology represents an excellent tool for testing this physics: the agreement of the sound velocity in the models and in the “seismic Sun” (deduced from helioseismic modes) is much better when element settling is introduced (e.g. Bahcall & Pinsonneault 1995, Richard et al. 1996, Brun et al. 1998, Vauclair 1998).

This is the reason why the lithium plateau becomes a paradox, with observations in contradiction with the predictions of the best available theory. What does the lithium plateau want to tell us that we have not yet understood?

The clue may be related to the physics of rotation-induced mixing in the presence of μ-gradients. This has been studied by several authors, beginning with Mestel (1953, 1957, 1961, 1965) and Kippenhahn (1958, 1963), and recently by Zahn (1992) and Maeder & Zahn (1998). In the present paper, the physics of meridional circulation in the presence of μ-gradients is revisited. We show that, when one takes into account helium gravitational settling, μ-induced currents opposing the traditional meridional currents (hereafter called Ω-currents) take place, and that they can become of the same order of magnitude. Then the circulation freezes out. Furthermore, in this case the element settling itself may also be prevented. We are in the presence of a self-regulating process where the rotational currents and the helium settling cancel each other. As lithium diffuses in a way similar to helium (at the same rate), the lithium settling is also cancelled by this process. We show how this could account for the small dispersion of the lithium plateau. It may also have other consequences which will be studied in forthcoming papers.

In Sect. 2, the theory of meridional circulation including the effects of μ-gradients is recalled. Orders of magnitude for the case of Pop II plateau stars are discussed in Sect. 3 and evidences of the importance of μ-induced currents are given. We show how element settling can be prevented when μ-induced currents cancel Ω-currents. A general discussion is presented in Sect. 4.

2. Meridional circulation including μ-induced currents

2.1. Classical meridional circulation

In rotating stars, the equipotentials of “effective gravity” (including the centrifugal acceleration) have ellipsoïdal shapes while the energy transport still occurs in a spherically symetrical way. The resulting thermal imbalance must be compensated by macroscopic motions: the so-called “meridional circulation” (Von Zeipel 1924). The stellar regions outside the convective zones cannot be in complete radiative equilibrium. They are subject to entropy variations given by (see the review by Zahn 1993):

$$\rho T\left(\frac{\partial S}{\partial t}+\mathbf{u}\cdot\nabla S\right) = -\nabla\cdot\mathbf{F}+\rho\varepsilon_n = \rho\varepsilon_\Omega\ (\neq 0) \quad (1)$$

where $\mathbf{F}$ represents the heat flux, ε_n the nuclear energy production and ε_Ω an energy generation rate which results from sources and sinks of energy along the equipotentials.

The vertical component of the meridional velocity u_r is computed as a function of ε_Ω in the stationary regime (from Eq. 1):

$$u_r=\left(\frac{P}{C_p\rho T}\right)\frac{\varepsilon_\Omega}{g} \quad (2)$$

which, for a perfect gas, reduces to:

$$u_r=\frac{\varepsilon_\Omega}{g}\ \frac{\nabla_{\rm ad}}{\nabla_{\rm ad}-\nabla+\nabla_\mu} \quad (3)$$

where g represents the local gravity, $\nabla_{\rm ad}$ and ∇ the usual adiabatic and real ratios $\left(\frac{d\ln T}{d\ln P}\right)$ and ∇_μ the mean molecular weight contribution $\left(\frac{d\ln\mu}{d\ln P}\right)$.

The expression of ε_Ω is computed by expanding the right-hand-side of Eq. (1) on a level surface and writing that its mean value vanishes.

For nearly-uniform rotation and negligible μ-gradients, one finds (cf Mestel 1965, Zahn 1993 and references therein):

$$\varepsilon_\Omega=\frac{8}{3}\ \frac{L}{M}\left(\frac{\Omega^2 r^3}{GM}\right)\left(1-\frac{\Omega^2}{2\pi G\rho}\right)P_2(\cos\theta) \quad (4)$$

all quantities being computed at radius r.

In this case, neglecting ∇_μ in (3), the vertical meridional velocity may be written:

$$u_r=\frac{8}{3}\ \frac{L}{M}\ \frac{\nabla_{\rm ad}}{\nabla_{\rm ad}-\nabla}\ \frac{\Omega^2 r}{g^2}\left(1-\frac{\Omega^2}{2\pi G\rho}\right)P_2(\cos\theta) \quad (5)$$

Introducing U_r as:

$$u_r=U_r\ P_2\ (\cos\theta) \quad (6)$$

the horizontal meridional velocity is given by (see Mestel 1965):

$$u_\theta = -\frac{1}{2\rho r}\,\frac{d}{dr}\,(\rho r^2 U_r)\sin\theta\cos\theta \tag{7}$$

In the presence of mean molecular weight gradients, these expressions must be modified, as discussed below.

2.2. μ-induced currents

Mestel (1953) and (1957) (see also Mestel 1965) pointed out that, in the presence of vertical μ-gradients, ε_Ω contains other terms related to the resulting horizontal variations of μ: the so-called "μ-induced currents". The expression of ε_Ω obtained in this case, with the assumption of perfect gas law, has been derived in detail.

More recently Maeder & Zahn (1998), hereafter MZ98, gave a modified expression for ε_Ω which takes into account several effects which were not included in the previous computations: more general equations of state instead of perfect gas law, presence of a thermal flux induced by horizontal turbulence, non-stationary cases.

Here we wish to focus on the importance of non-negligible μ-gradients for the meridional circulation, which may modify our understanding of the low-rotation regimes in the presence of microscopic diffusion.

The basic idea is that on a level surface (with constant pressure) specific quantities like temperature T, density ρ, gravity g, thermal conductivity χ, rotation rate Ω and mean molecular weight μ may vary around an average value. Following Zahn 1992 and MZ98 these quantities x are expanded on isobars as:

$$x(P,\theta) = \overline{x}(P) + \tilde{x}(P)P_2(\cos\theta) \tag{8}$$

The expansion of the energy term in Eq. (1) leads to derivatives in $\frac{d}{dP}$ which have to be transformed through an equation of state to lead to the radius derivatives $\frac{d}{dr}$.

In this procedure, two kinds of terms finally appear:

1) those which are directly related to the rotation rate, for example the gravity fluctuations

2) those which are related to the local μ-gradient.

As we discuss a basic physical point, we prefer to simplify the expressions under specific assumptions, neglecting for the moment the secondary terms. We assume perfect gas law, and nearly rigid rotation (the importance of this assumption will be discussed in Sect. 4). In this case, the gravity fluctuations are given by:

$$\frac{\tilde{g}}{g} = \frac{4}{3}\left(\frac{\Omega^2 r^3}{GM}\right) \tag{9}$$

As we focus on the physical processes which occur below the convection zones in cool stars, we also neglect all the terms related to energy production (stellar cores will be discussed in a forthcoming paper). Keeping only the non-negligible terms in MZ98's expressions, we obtain:

$$\varepsilon_\Omega = \left(\frac{L}{M}\right)(E_\Omega + E_\mu)\,P_2(\cos\theta) \tag{10}$$

with:

$$E_\Omega = \frac{8}{3}\left(\frac{\Omega^2 r^3}{GM}\right)\left(1 - \frac{\Omega^2}{2\pi G\overline{\rho}}\right) \tag{11}$$

$$E_\mu = \frac{\rho_m}{\overline{\rho}}\left\{\frac{r}{3}\,\frac{d}{dr}\left[\left(H_T\frac{d\Lambda}{dr}\right) - (\chi_\mu + \chi_T + 1)\Lambda\right] - \frac{2H_T\Lambda}{r}\right\} \tag{12}$$

Here $\overline{\rho}$ represents the density average on the level surface ($\simeq \rho$) while ρ_m is the mean density inside the sphere of radius r; H_T is the temperature scale height; Λ represents the horizontal μ fluctuations $\frac{\tilde{\mu}}{\overline{\mu}}$; χ_μ and χ_T represent the derivatives:

$$\chi_\mu = \left(\frac{\partial\ln\chi}{\partial\ln\mu}\right)_{P,T} \quad ; \quad \chi_T = \left(\frac{\partial\ln\chi}{\partial\ln T}\right)_{P,\mu} \tag{13}$$

In the following we will also neglect the "Gratton-Öpik" term $\frac{\Omega^2}{2\pi G\rho}$, which is justified below the convective zones in slowly-rotating cool stars.

2.3. Physical interpretation

In the presence of μ-gradients, the circulation velocity is the sum of two terms: E_Ω, which we shall call the "Ω-current", and E_μ, which we shall call the "μ-induced current" or, for short, the "μ-current". In the general case, E_μ may also be related to Ω through the horizontal μ-fluctuations Λ. We will see however that, in the stationary case, Λ is independent of Ω. In most of the star, E_Ω is positive (except in the very outer layers) while E_μ is negative (except in case of strong second derivatives of Λ, which are supposed negligible in the following): the μ-currents are opposite to the Ω-currents. When Λ varies directly like r (this situation occurs in the stationary case, as will be seen below), E_μ becomes:

$$E_\mu = -2\,\frac{\rho_m}{\rho}\,\frac{H_T}{r}\Lambda\left[1 + \frac{(1+\chi_T+\chi_\mu)}{6}\,\frac{r}{H_T}\right] \tag{14}$$

Which we will write:

$$E_\Omega \simeq \frac{8}{3}\,\frac{\Omega^2 r^3}{GM} \tag{15}$$

$$E_\mu \simeq -c_\Lambda\,\frac{2\rho_m}{\rho}\,\frac{H_T}{r}\Lambda \tag{16}$$

where c_Λ is of order unity.

The physical interpretation of these currents can be given in the following hand-waving way. Due to centrifugal effects, the effective gravity is constant on ellipsoidal shells (Fig. 1) while the temperature gradients keep the spherical symetry. Consequently the temperature is larger at the poles than at the equator on a level surface, leading to the Ω-currents. The element abundances are also expected to vary along level surfaces, as they depend on temperature (in case of gravitational settling, they also depend on gravity, so that they should not be constant on spheroids either).

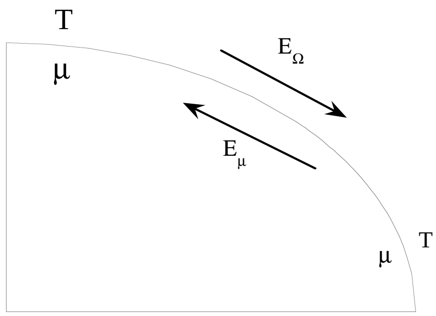

Fig. 1. Schematic drawing of a "level surface" in a rotating star. Temperature and mean molecular weight are larger at the poles than at the equator, inducing a competition between two effects: the so-called "Ω-currents" from pole to equator and "μ-currents" from equator to pole. At equilibrium, the circulation stops. Then element settling proceeds at the pole in a more rapid way than at the equator, thereby decreasing the horizontal μ-gradient and $E\mu$. The circulation begins again with matter going up at the pole: it brings back the material in which helium has settled, restablishing the equilibrium horizontal μ-gradient. Such a process can cancel altogether meridional circulation and helium settling.

The density fluctuations on level surfaces are given by (Zahn 1992):

$$\frac{\tilde{\rho}}{\rho} = \frac{1}{3}\,\frac{r^2}{\overline{g}}\,\frac{d\Omega^2}{dr} \tag{17}$$

As a consequence, in the approximation of nearly rigid rotation, level surfaces are both surfaces of equal pressure and density. In this case μ varies like T.

In first approximation, all other parameters assumed constant, the specific entropy is proportional to the number of particules per unit volume (Sackur-Tetrode formula, see Vauclair 1993), or inversely proportional to μ. Thus the horizontal μ-gradients lead to a larger entropy at the equator than at the pole, which explains why μ-currents are opposite to Ω-currents.

The μ-currents induced by the flattening of the level surfaces are very small though, so that they do not prevent the onset of the meridional circulation. However, as soon as the circulation begins, the horizontal μ-gradients increase as the upward flows bring μ-enriched matter at the poles while the downward flows bring μ-depleted matter at the equator. As will be seen below, a stationary solution can occur for which, in some cases, the μ-currents may become of the same order as the Ω-currents, leading to new interesting physics.

2.4. Discussion on the horizontal variations of μ

The order of magnitude of the horizontal μ-gradients induced by the flattening of the level surfaces Λ_f can be obtained with the approximation that the radius differences between the polar and equatorial regions are given by:

$$\lambda = \frac{\Delta r}{r} \simeq \frac{\Omega^2 r^3}{GM} \tag{18}$$

The values of this flattening parameter are given in Fig. 2 as a function of r in a 0.8 $M_\odot$ halo star which rotates with a velocity of 10 km.s^{-1}.

In this case, the horizontal μ fluctuations can be written:

$$\Lambda_f = \frac{\tilde{\mu}}{\mu} \simeq \frac{\Omega^2 r^4}{GM} \cdot \nabla \ln \mu \tag{19}$$

or, with the following definition of the "μ-scale height":

$$H_\mu = (\nabla \ln \mu)^{-1} \tag{20}$$

the horizontal fluctuations related to the flattening become:

$$\Lambda_f = \frac{\Omega^2 r^3}{GM}\,\frac{r}{H_\mu} \tag{21}$$

These fluctuations are small compared to those which occur as soon as the meridional circulation is established. In case of a laminar circulation (pure advection) the order of magnitude of Λ is given by:

$$\Lambda_a \simeq r \cdot \nabla \ln \mu \simeq \frac{r}{H_\mu} \tag{22}$$

According to Zahn (1992) and Chaboyer & Zahn (1992), the shears induced by the circulation lead to a strong horizontal turbulence, which reduces the horizontal μ-gradients. In this case, introducing a horizontal diffusion coefficient D_h, the local variations of μ are solutions of:[1]

$$\frac{\partial \tilde{\mu}}{\partial t} + u(r) \cdot \frac{\partial \overline{\mu}}{dr} = -\frac{6}{r^2}\, D_h\, \tilde{\mu} \tag{23}$$

In the stationary case, we obtain for the horizontal μ-gradient including turbulence:

$$\Lambda_t = \frac{\tilde{\mu}}{\overline{\mu}} = -\frac{U_r \cdot r^2}{6 D_h}\,\frac{\partial \ln \mu}{\partial r} \tag{24}$$

With the assumption that D_h is proportional to $U_r \cdot r$ and not simply to U_r as written by mistake in MZ98, Eq. (22) becomes:

$$\Lambda_t \simeq -\frac{1}{6\alpha_h}\,\frac{\partial \ln \mu}{\partial \ln r} \simeq -\frac{1}{6\alpha_h}\,\frac{r}{H_\mu} \tag{25}$$

Replacing in Eq. (16) gives the simple expression:

$$E_\mu = -\frac{1}{3\alpha_h}\,\frac{H_T}{H_\mu}\,\frac{\rho_m}{\overline{\rho}} \tag{26}$$

with $\overline{\rho} \simeq \rho$ and $\rho_m = \dfrac{3M}{4\pi r^3}$

In summary, in the most likely case of meridional circulation associated with a strong horizontal turbulence, the vertical velocity of meridional circulation can simply be written:

$$\begin{aligned} u_r &= U_r P_2(\cos\theta) \\ \text{with} \quad U_r &= \frac{1}{g}\left(\frac{\nabla_{\rm ad}}{\nabla_{\rm ad} - \nabla + \nabla_\mu}\right)\left(\frac{L}{M}\right)(E_\Omega + E_\mu) && (27) \\ E_\Omega &= \frac{8\Omega^2 r^3}{3GM} && (28) \\ E_\mu &= -\frac{C_\Lambda}{\alpha_h}\,\frac{M}{4\pi r^3 \rho}\,\frac{H_T}{H_\mu} && (29) \end{aligned}$$

[1] Following Chaboyer & Zahn (1992) and MZ 98, μ is developped on r instead of P. The difference is negligible though, as the flattening effect on μ is extremely small.

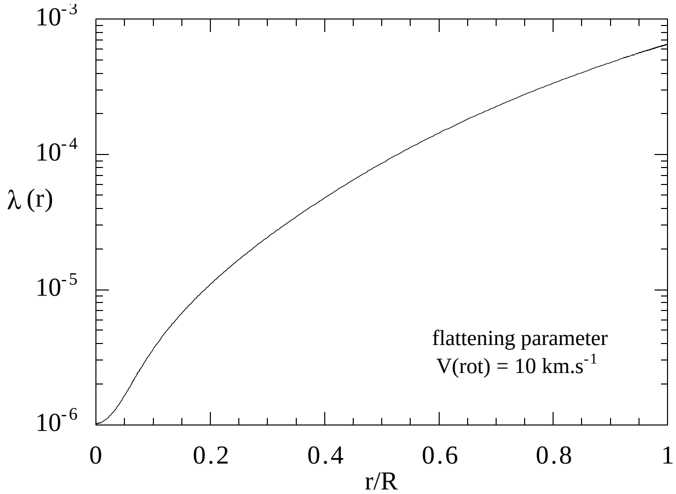

Fig. 2. Flattening parameter $\lambda = \dfrac{\Delta r}{r} \simeq \dfrac{\Omega^2 r^3}{GM}$ in a 0.8 $M_\odot$ plateau star with $V_{\rm rot} = 10$ km.s^{-1} (see text for details).

According to Zahn's (1993) prescriptions, while the horizontal diffusion coefficient is larger than $U_r \cdot r$ $(D_h = \alpha_h U_r \cdot r$ with $\alpha_h > 1)$, the vertical transport may be approximated as an effective diffusion coefficient of order:

$$D_{\rm eff} \simeq \frac{1}{C_h} U_r \cdot r \qquad \text{with} \qquad C_h \simeq 30$$

The reason why E_μ does not depend on Ω but depends on H_μ^{-1} may be physically interpreted in the following way. Suppose that at level r the meridional circulation can be approximated by an upwards and a downwards moving flows. The horizontal turbulence, described by the horizontal turbulent diffusion coefficient D_h, leads to matter transfer from one flow to the other. In this framework, a vertical μ-gradient induces a horizontal μ-gradient which plays a non-negligible role in the entropy balance and has a feed-back effect on the velocity U_r. Increasing the rotation velocity should increase U_r, the horizontal μ-gradient and the corresponding D_h. But the increase of the mixing effect related to D_h decreases this horizontal μ-gradient. The horizontal turbulence "swallows" the possible variations induced by different rotation velocities, leaving the vertical mixing process unchanged.

3. Orders of magnitude and discussion

3.1. Comparison between the Ω-currents and the μ-currents

At this point, it is interesting to compare the orders of magnitudes of E_Ω and E_μ in stellar models. As the present paper has been motivated by the existence of the lithium plateau in halo stars, we chose to discuss as an example the case of a 0.8 $M_\odot$ low metallicity star, situated at the hot end of the plateau ($Z = 10^{-3}$; age: 12 Gyr). Fig. 2 gives the values of the flattening parameter $\lambda(r) = \dfrac{\Omega^2 r^3}{GM}$, with the assumption of a stellar rotation velocity $V_{\rm rot} = 10$ km.s^{-1} which represents an upper limit for these slowly rotating stars. Fig. 3 displays the abundance profile in the same star, including element settling (Charbonnel & Vauclair 1999). It also shows the μ-gradient $\nabla \ln \mu$, which is essentially related to the helium profile.

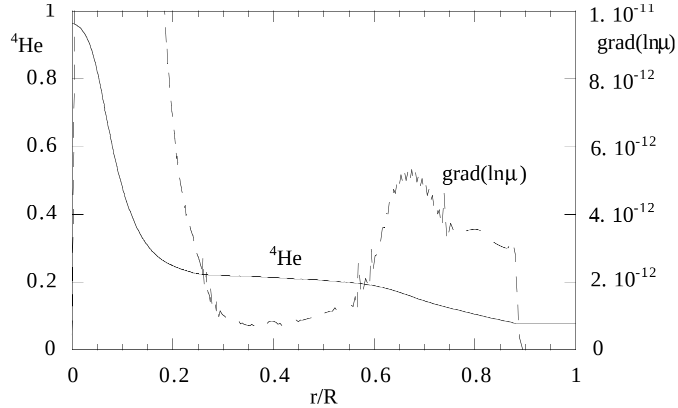

Fig. 3. Helium abundance profile and grad($\ln \mu$) in a 0.8 $M_\odot$ plateau star, including element settling, at the age of 12 Gyr. The increase in grad($\ln \mu$) below the convective zone is clearly related to the ^{4}He slope.

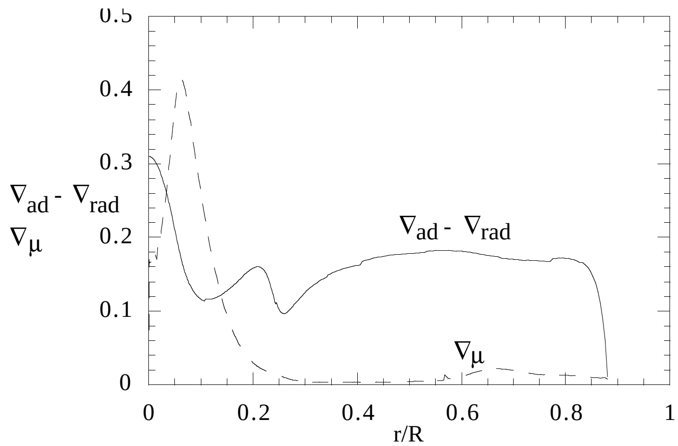

Fig. 4. Comparison of the μ-gradient $\dfrac{d \ln \mu}{d \ln P}$ with the usual adiabatic and radiative gradient differences. The μ-gradient is always negligible compared to the others, except in the core and right at the bottom of the convective zone.

The values of $\nabla_\mu = \left(\dfrac{\partial \ln \mu}{\partial \ln P}\right)$ are given in Fig. 4 also for the model including element settling, together with the difference $\nabla_{\rm ad} - \nabla$: we can see that ∇_μ is always negligible except in the central core and right at the bottom of the convective zone. The μ-scale heights H_μ, defined as $\left(\dfrac{d \ln \mu}{dr}\right)^{-1}$ are given in Fig. 5 for two models of the same star, one without element settling (st) and the one including element settling (dif). The temperature scale height H_T, computed in the model with element settling, is also shown (the difference in H_T between the two models is negligible). The effect of helium diffusion is clearly visible on this graph.

Fig. 6 presents for these two models a comparison of the absolute values of E_μ and E_Ω. Here again the rotation velocity is taken as 10 km.s^{-1} which represents an upper limit for halo stars. The E_μ values are computed with the assumption of

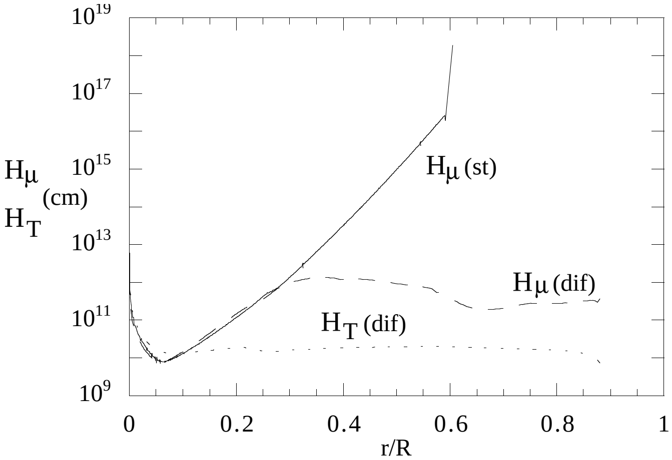

Fig. 5. μ-scale heights computed in two different 0.8 $M_\odot$ plateau stars models: one with no element settling included (st) and one with element settling included (dif). The temperature scale-height is also shown for the model with element settling (there is no significant difference with the other model). The difference in the μ-scale heights is the signature of helium settling.

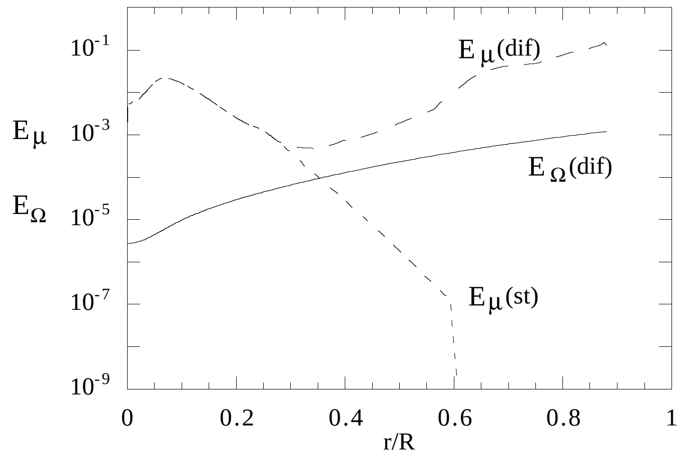

Fig. 6. μ-currents (in absolute values) computed in the two 0.8 $M_\odot$ plateau stars models, with and without element settling, and Ω-current computed in the model with settling, assuming $V_{\rm rot} = 10$ km.s^{-1} and $\alpha = 10$ (see text). We can see that in the model with settling, $|E_\mu|$ is always larger than $|E_\Omega|$. As most halo stars have rotation velocities below this value, $|E_\Omega|$ is expected to be even smaller than shown here, which reimforces this conclusion.

meridional circulation and horizontal turbulence, and the diffusion coefficient D_h is assumed equal to $10\ U_r \cdot r$ ($\alpha_h = 10$). We can see that in the model without settling, $|E_\mu|$ is larger than $|E_\Omega|$ in the central regions and smaller in the outer layers while in the model including settling, $|E_\mu|$ is always larger than $|E_\Omega|$. This means that some physical process must occur which has not yet been included in the theory. The case of the stellar core, where other effects can occur, will be discussed in a forthcoming paper. Here we focus on the radiative region below the convective zone.

Suppose the star begins on the main sequence with homogeneous abundances. Then $|E_\mu|$ lies below $|E_\Omega|$ and the meridional circulation can occur in the same way as we have already discussed. Horizontal turbulence may develop, and a vertical mixing takes place with an effective diffusion coefficient $D_{\rm eff}$ (Zahn 1992). This vertical mixing slows down the element settling without stopping it. Defining H_c as the concentration scale height, the element settling would stop only for:

$$\frac{D_{\rm eff}}{H_c} \simeq \frac{k_p D_m}{H_p} \tag{30}$$

where D_m is the microscopic diffusion for the considered element and k_p the coefficient in front of the pressure gradient in the microscopic diffusion velocity (see, for example, Vauclair & Vauclair 1982). However, before reaching this equilibrium concentration, **the induced μ-currents become of the same order as the Ω-currents**, therefore stopping the circulation.

We can compute, for the same 0.8 $M_\odot$ star, the critical μ-scale height for which $|E_\mu| = |E_\Omega|$.

We obtain:

$$H_\mu^{\rm crit} = \frac{H_T}{8\alpha_h \lambda}\ \frac{\rho_m}{\rho} = \frac{H_T}{\alpha_h}\ \frac{3GM^2}{32\pi\Omega^2\rho r^6} \tag{31}$$

The profile of $H_\mu^{\rm crit}(r)$ is given in Fig. 7, also for $V_{\rm rot} = 10$ km.s^{-1} and $\alpha_h = 10$. For different rotation velocities $H_\mu^{\rm crit}$ is simply multiplied by $(100/v_{\rm rot})^2$.

It is interesting here to compare this critical μ-scale height with the concentration scale height given by Eq. (30). For a simple mixture of completely ionised hydrogen and helium, the mean molecular weight is given by:

$$\mu \simeq \frac{1+4c}{2+3c} \tag{32}$$

With c of the order of 0.1, we find for helium:

$$H_\mu \simeq 6 H_c \tag{33}$$

Eq. (30) gives, for the equilibrium concentration scale height:

$$H_c = H_p \frac{D_{\rm eff}}{k_p D_m} \tag{34}$$

Taking $k_p \simeq 5$, $H_p \simeq 10^{10}$ and $D_{\rm eff}/D_m \simeq 100$ we find:
$H_c \simeq 2 \times 10^{11}$ cm and $H_\mu \simeq 10^{12}$ cm.

We can check from Fig. 7 that the critical H_μ which stops the circulation is reached much before the equilibrium H_c which would stop the helium settling.

Now the question arises: what happens when $|E_\mu|$ becomes equal to $|E_\Omega|$?

3.2. Self-regulating process in case of meridional circulation and settling

Let us first summarize the situation of a slowly rotating star in which element settling leads to an increase of the μ-gradient below the outer convection zone.

At the beginning, the star is homogeneous and meridional circulation can occur, leading to upward flows in the polar regions and downward flows in the equatorial parts (except in the

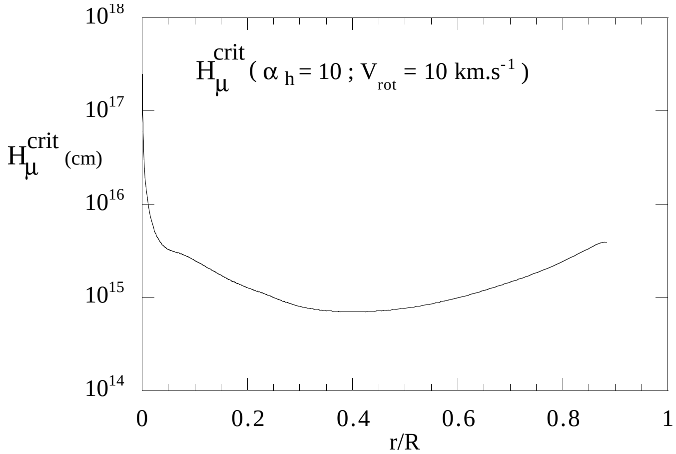

Fig. 7. Critical μ-scale height for which $|E_\mu| = |E_\Omega|$ in a 0.8 $M_\odot$ plateau star, with the same assumptions as in Fig. 6. In this case, the μ-scale height is more than 5 orders of magnitude larger than the stellar radius: the resulting helium (and also lithium) profiles should be flat below the convection zone. For lithium, it should remain flat down to the nuclear destruction region. As $H_\mu^{\rm crit}$, varies like $(v_{\rm rot})^{-2}$, this conclusion is reimforced for slower rotations.

very outer layers where the Gratton-Öpik term becomes important, which we do not discuss here). The μ-currents, opposite to the classical Ω-currents, are first negligible. The μ-gradients increasing with time because of helium settling, the order of magnitude of the μ-currents also increase and the μ-scale height decreases until it reaches $H_\mu^{\rm crit}$ for which the circulation vanishes.

This does not occur all at once: as $H_\mu^{\rm crit}$ decreases with depth below the convective zone (Fig. 7) we expect that the meridional circulation freezes out step by step. An equilibrium situation may be reached, in which the temperature and mean molecular weight gradients along the level surfaces are such that Ω-currents and μ-currents cancel each other.

Once it is reached, this equilibrium situation is quite robust. Suppose that some mechanism leads to a decrease of the horizontal μ-gradient: then $|E_\mu|$ becomes smaller then $|E_\Omega|$ and the circulation tends to be restablished in the $|E_\Omega|$ direction. The flow goes up in the polar regions, bringing back matter with larger μ: the μ gradient is restored and the circulation stop.

Suppose now that the horizontal μ-gradient is increased. Then $|E_\mu|$ becomes larger than $|E_\Omega|$ and the circulation begins in the E_μ direction. Matter with larger μ goes up in the equatorial region, decreasing the μ-gradient. Here again the circulation stops.

As there is no more circulation, we would expect that helium settling can proceed further. The microscopic diffusion velocity, in case of pure gravitational settling, can simply be written (Vauclair & Vauclair 1982):

$$V_D = -D_m \frac{k_p}{H_p} = -D_m k_p \frac{\mu g}{kT} \qquad (35)$$

with:

$$D \;\alpha\; T^{5/2}/\rho \qquad (36)$$

On a level surface, μ varies like T, so that $v_D \;\alpha\; T^{5/2}$ and $\dfrac{\Delta v_D}{v_D} = \dfrac{5}{2}\dfrac{\Delta T}{T} = \dfrac{5}{2}\Lambda$.

As a consequence, on a level surface, the helium gravitational settling is more efficient in the regions of larger T, namely the polar regions, than in the equatorial regions.

We can now describe the scenario: once the meridional circulation is frozen, the helium gravitational settling would like to proceed and lead to a decrease of the horizontal μ-gradient. However the μ-currents become smaller than the Ω-currents and μ-enriched matter is brought back into the polar region, restoring the original μ-gradient.

We may be here in the presence of a self-regulating process in which the μ-gradients which should increase with time due to the settling are prevented to do so because of the currents equilibrium. Such a process would stop altogether the meridional circulation **and** the element settling!

In this case the element abundances that we observe at the surface would be the result of the self-regulating process and not the result of pure settling. For halo stars, as shown on Fig. 7, this would occur for H_μ much larger than the stellar radius: the abundance profiles would be flat below the convection zone.

This process could be the reason for the low dispersion of the lithium abundance in the lithium plateau of halo stars: the lithium abundance profile would be flat down to the place where the diffusion time scale is of the order of the nuclear destruction time scale, that is the place of the lithium peak (Vauclair & Charbonnel 1998). The observed abundance would then depend only on the microscopic physical parameters, constant from star to star, which would explain the very small observed dispersion. This process will be studied numerically in a forthcoming paper.

4. Conclusion

Computations of meridional currents in the presence of μ-gradients show that, as already pointed out long ago by Mestel (1953), and recently studied by Zahn (1992) and Maeder & Zahn (1998), μ-induced currents (μ-currents in short) settle in opposition to Ω-currents. In the present paper, we have computed the orders of magnitude of these currents under simplifying assumptions. Among these assumptions (energy production negligible, perfect gas equation of state, Gratton-Öpik term negligible, nearly solid rotation), the most stringent one is the assumption of nearly-solid rotation, as pointed out by Zahn (1999). Assuming solid rotation means that some other process like internal waves must have forced the transport of angular momentum (Zahn et al. 1997). In any case, we know from helioseismology that the Sun does indeed rotate as a solid body below the outer convective zone (Brown et al. 1989). This is not yet quite understood, but we may infer that the same process which forces solid rotation inside the Sun also acts in other solar-type stars. This may not be crucial for our general conclusions, although including differential rotation would need a more complicate treatment of the equations, and would add another unknown parameter. This may be studied in the future.

We have shown that under these assumptions μ-currents may become of the same order of magnitude as Ω-currents in case of element settling, long before the concentration gradient induced in case of pure settling is reached. In the case of a 0.8 $M_\odot$ halo star with standard helium settling, we have computed that the μ-currents would be several orders of magnitude larger than the Ω-currents! This is also the case in the central stellar regions, due to nuclear reactions.

Here we have only focused on the physical processes which occur below the outer convective zones. In the case of halo stars we found that, for slow rotation, μ-currents cancel Ω-currents for very small concentration gradients, of order $10^{-15}(H_\mu^{\rm crit} = 10^{15}$ cm). Furthermore, in this case helium setlling may also be cancelled out as it would decrease the effect of horizontal μ-gradients, thereby setting again meridional currents which would restore the original gradients. This may become a self-regulating process which could explain why the observed lithium in the plateau stars has such a small dispersion: the inferred lithium gradient inside the star, below the convective zone, would be very small and the lithium profile nearly flat until the lithium nuclear destruction region. This would give a surface lithium abundance close to that of the "lithium peak" (Vauclair & Charbonnel 1998), the same one for all the stars.

Numerical simulations of the whole process have to be done, to check all these effects. It will also be tested for the solar case (including central regions), and for the case of Am stars (for which the Gratton-Öpik term is not negligible), in forthcoming papers. There are many observations in stars which give evidences of mixing processes occuring below the outer convection zones as, for example, the lithium depletion observed in the Sun and in galactic clusters. The process we have described here should not apply in all these stars. The reason could be related to the rapid rotation of young stars on the ZAMS and to their subsequent rotational braking (here we suppose that the stars always rotated slowly). Also for stars in which the Gratton-Öpik term is no more negligible, the whole process has to be revised. Here we have only studied equilibrium situations, while time-evolving processes should be tested in the future. Finally other mixing processes, like ABCD or GSF instabilities, could play a role.

In any case the present computations show that element settling in slowly rotating stars leads to surface abundances which depend on the competition between μ-currents and Ω-currents. This may change our general understanding of the diffusion processes of chemical species in rotating stars.

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