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Clarification and interpretation of the Friedjung-Muratorio self absorption curve method

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Abstract. The Self Absorption Curve (SAC) method of Friedjung and Muratorio is derived directly from a more usual description of emission from optically thick sources. The present derivation shows that the SAC function $Q(t_0)$ is the logarithm of the line-of-sight photon escape probability associated with the particular model used. A description of the photon escape probability concept and its present state of development is therefore given, with some examples of source models currently utilized.

Key words: line: identification – radiative transfer

1. Introduction

For the analysis of spectral lines emitted from dense sources such as quasars and luminous blue stars, which have appreciable self-absorption (i.e. opacity), Friedjung & Muratorio (1987) (FM) have developed a useful semi-empirical method involving what they term a self-absorption curve (SAC). The method as described by FM is however not transparent in its formulation or application, leading Friedjung (1988) to comment "Unfortunately the method (has) not been sufficiently studied by colleagues". The present communication aims to clarify the FM analysis and make it more accessible by relating it to, and interpreting it in terms of, already known treatments of optically thick line emission in the literature. In particular it is shown that the SAC function $Q(\tau_c)$ is directly related to the photon escape probability concept, which is therefore described in some detail, with associated aspects which are relevant for application of (i.e. modelling by) the SAC method.

2. Analysis

2.1. The FM SAC method

The essential analysis of the SAC method will first be described. For a given emission line i of wavelength λ_i and oscillator strength f_i, with lower level statistical weight g_i, and of equivalent width $W(\lambda_i)$ referred to a continuum flux $F(\lambda_i)$, the quantity $\log[F(\lambda_i)W(\lambda_i)\lambda_i^3/g_i f_i]$ is plotted as an ordinate $\log y_i$ against the quantity $\log(g_i f_i \lambda_i)$ as an abscissa $\log x_i$, giving a corresponding point in the ($\log x$, $\log y$) plane. FM showed that the ordinate $\log y_i$ is equivalent to:

$$\log\left[\frac{F(\lambda_i)W(\lambda_i)\lambda^3}{g_i f_i}\right] = \log(2k\pi hcV_c{R_c}^2/d^2) + \log(\phi_{ci}) + Q(\tau_c) \quad (1)$$

where R_c, τ_{ci} and V_c are characteristic values of the source radius, the line optical thickness (at line-center) and the linewidth in velocity units, with d the distance to the source. The constant $k = 0.02654\,\mathrm{cm}^2\,\mathrm{s}^{-1}$ (k here is not the Boltzmann constant; this can be confusing). The quantity $\phi_c^i \equiv (1/V_c)\int_z (N_{ui}/g_{ui})dz$ is a normalized source column density of the upper level u. The function $Q(\tau_c)$ is the essential SAC function which defines the dependence of the left-hand-side of (1) on the line opacity.

The ordinate $\log y_i$ thus depends on optical depth τ_{ci}, through the opacity function $Q(\tau_c)$, and on the parameter of upper level population N_{ui}, through the quantity ϕ_{ci}^i.

On the other hand, the abscissa quantity $\log x \equiv \log(gf\lambda)$ is directly related to τ_c by [expression (4) of FM]:

$$\tau_c = (kgf\lambda/V_c)\int_z \frac{N_u}{g_u}dz \quad (2)$$

so that

$$\log gf\lambda = \log\tau_c - \log\left[int_z\frac{N_l}{g_l}dz\right] + \log\ V_c - \log k \quad (3)$$

and thus the abscissa $\log x_i$ depends on the line optical thickness τ_{ci} and on the parameter of lower level population density N_{li}.

A given emission line i is thereby represented by a point ($\log x_i$, $\log y_i$) on the ($\log x$, $\log y$) plot for which x_i is determined by the line's lower level population density N_{li} and its line-center optical thickness τ_{ci}, and y_i is determined by the line's upper level population density N_{ui}.

As described in Muratorio et al. (1992), in practice the method is applied to spectra comprising several multiplets of a given atom or ion. Points (x_i, y_i) corresponding to component lines of each multiplet, obtained through observations of their equivalent widths $W(\lambda_i)$, are fitted to a model SAC curve for that multiplet, i.e. a model function $Q(\tau_c)$. Subsequently all the multiplet curves are horizontally and vertically shifted with reference to a chosen reference multiplet in order to determine the

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relative level population distribution among the emitting atom's levels, and at the same time to derive the "global" SAC curve, i.e. the best-fitting (empirical) $Q(\tau_c)$ function.

2.2. Alternative derivation of the SAC method

The above analysis and its result may however be derived more directly, starting from a more usual description. The power radiated from an optically thin uniform source is given by:

$$N_u h\nu A_{ul} V \equiv N_u \frac{hc}{\lambda} A_{ul} \pi R^2 L \quad \mathrm{erg\,s^{-1}} \tag{4}$$

where N_u is the upper level population density (cm^{-3}), A_{ul} is the radiative rate (s^{-1}) and $V = \pi R^2 L(\mathrm{cm}^3)$ is the source volume for a source of characteristic radius R and length L. At the observer, at distance d from the source, the flux received is:

$$\frac{N_u}{4\pi}\frac{hc}{\lambda}A_{ul}\frac{\pi\, R^2\, L}{d^2} \equiv \frac{(0.6670)}{\lambda^3}\frac{\pi hc}{4\pi}\frac{R^2}{d^2} \times L N_u f_{lu}\frac{g_l}{g_u} \quad \mathrm{erg\,cm^{-2}\,s^{-1}} \tag{5}$$

using $A_{ul} = (0.6670/\lambda^2)(g_l/g_u)f_{lu}$. This is the flux $F(\lambda)W(\lambda)$ of FM. Multiplying it by the factor $(\lambda^3/g_l f_{lu})$ one obtains:

$$F(\lambda)W(\lambda)\,\frac{\lambda^3}{g_l f_{lu}} = (0.05313)\frac{\pi hc}{g_u}\,\frac{R^2}{d^2}\,N_u\,L = \frac{2k\pi hc}{g_u}\frac{R^2}{d^2}N_u\,L \tag{6}$$

in essential agreement with FM's expression (7), when the column density $N_u L$ of the uniform source is replaced by $\int_z N_u dz$ for a non-uniform source.

For an optically thick source of line-center optical thickness τ_0, however, expression (4) must further be multiplied by an appropriate monodirectional photon escape probability $p(\tau_0)$. The equivalence of (6), when multiplied by $p(\tau_0)$, and the final FM expression (11) demonstrated here shows that FM's general SAC function $Q(\tau_c)$ is actually $\log[p(\tau_c)]$. The latter function is well-known for simple source models. For example the SAC function $Q_1(\tau_c)$ of FM's expression (12), appropriate for a uniform medium emitting a Doppler-broadened line, is (apart from a different normalization, i.e. definition, of optical thickness τ) just $\log[p_f(D, \tau_0; 1)]$ in the notation of Kastner & Kastner (1990) (KK); here D denotes Doppler line profile, and the Bartels parameter $q = 1$ signifies a uniform medium, i.e. a medium with a spatially constant source function $S_\nu = \varepsilon_\nu/k_\nu$. The subscript f further denotes that this particular quantity is a single flight or free-flight escape probability, implying that scattered photons do not escape.

Note also that the monodirectional escape probability $p_f(\tau_0)$ thus entering into this analysis is not to be confused with the directionally-averaged escape factor of the source. The reader is referred to a review (Kastner 1994) which compares and differentiates between the different types of escape probability.

3. Discussion of the SAC function/escape probability function

It is evident that the accuracy of results obtained by the FM SAC method will depend most critically on the appropriateness of the chosen SAC model function $Q(\tau_c)$, which has just been shown to be in general the logarithm of the photon escape probability quantity $p(\phi(\nu), \tau_c; S(t))$ for lines with frequency profiles $\phi(\nu)$ and for a source represented by a source function $S(t)$ where t is an optical depth into the source. A fuller description of this quantity may therefore be relevant, limited here to static sources which can be most easily described (FM deal with more complex cases of stellar wind emission). The discussion will further be limited to Doppler-broadened lines and to the single-flight escape situation, as opposed to the more general situation in which scattered photons can also escape. Thus singly-, doubly- and higher-order scattered photons are not considered, and the corresponding range of validity of the discussion is necessarily limited to low line-center optical thicknesses. Finally, the simplest possible spatial geometry is assumed, namely plane-parallel or "slab" geometry; other geometries have as yet been little investigated from the viewpoint of photon escape probabilities. These restrictions on the source model may seem narrow from the viewpoint of construction of general SAC functions, applicable also at higher optical thicknesses. However, only for such restricted models - and only recently - has the escape probability concept been quantitatively validated (Kastner 1999).

3.1. Photon escape probability

The emergent intensity $I(x, s; \tau_0)$ in a Doppler-profile line (x is the dimensionless frequency variable, for which FM use v) along a line-of-sight s is given by

$$I(x, s) = \int_0^{\tau_0} S(t)\ \exp(-x^2)\ \exp(-t\exp(-x^2))\,dt \tag{7}$$

where the optical depth $t = k_0 s$ and the total optical thickness of the source is $\tau_0 = k_0 s_0$, where s_0 is the length of the source along the line of sight; and where the source function $S(t)$ is appropriate to the geometry and the particular line-of-sight.

Assuming isotropic scattering, the emergent flux along the line-of-sight is then:

$$F(s; \tau_0) = 2\pi\int_0^1 \mu d\mu \int_{-\infty}^{\infty} I(x, s)dx = \pi\int_{-\infty}^{\infty} I(x, s)dx \quad \mathrm{erg\,cm^{-2}\,s^{-1}} \tag{8}$$

with $\mu = \cos(s.k)$, k being an arbitrary direction.

For the optically thin situation in which $\tau_0 \to 0$, the emergent intensity is

$$I(x, s; \tau \to 0) = \exp(-x^2)\int_0^{\tau_0} S(t)dt \tag{9}$$

so that the corresponding flux is:

$$F(s; \tau \to 0) = \pi\int_{-\infty}^{\infty}\exp(-x^2)dx\int_0^{\tau_0} S(t)dt$$

$$= \pi^{3/2} \int_0^{\tau_0} S(t)dt \quad (10)$$

The monodirectional photon escape probability is the ratio of the flux $F(s;\tau_0)$ emerging from the optically thick source to the flux $F(s;\tau_0 \to 0)$ that would emerge if the source were optically thin, and is therefore given by:

$$\begin{aligned} &p(D,\tau_0;S(t)) \\ &\quad \equiv \frac{F(s;\tau_0)}{F(s;\tau_0 \to 0)} \\ &\quad = \frac{\int_{-\infty}^{\infty} \exp(-x^2)dx \int_0^{\tau_0} S(t)\exp(-t\exp(-x^2))dt}{\sqrt{\pi}\int_0^{\tau_0} S(t)dt} \quad (11) \end{aligned}$$

This general expression for the photon escape probability thus contains, of course, the source function $S(t)$ as an essential factor.

As an example, the FM SAC function $Q_1(\tau_0)$ is obtained from (11) by assuming a spatially constant source function, i.e $S(t) = S$ a constant independent of location in the source. This yields directly

$$p(D,\tau_0;1) = \frac{1}{\sqrt{\pi}\tau_0} \int_{-\infty}^{\infty} \left[1 - \exp(-\tau_0 \exp(-x^2))\right] dx \quad (12)$$

which is the antilogarithm of $Q_1(\tau_0)$.

That $Q_1(\tau_0)$ assumes a constant source function has some important implications for its use. In particular it will lose validity for source optical thicknesses greater than about 10, when the source function departs from uniformity and self-reversal of the line profile begins to develop, even in a uniform medium of constant temperature. Thus curve b in FM's Fig. 2 will be inaccurate for $\log\tau_0 \gg 1$. On the other hand, for $\tau_0 < 10(\log\tau_0 < 1)$, $Q_1(\tau_0)$ should be a useful model function when applied to Doppler-broadened lines in static sources with negligible differential velocities. This has been verified by the writer in an application to a laboratory source spectrum (Kastner 1999).

The dependence of the escape probability $p_f(D,\tau_0;1)$ on τ_0 is shown in Fig. 1. It has a shape generally resembling the three-parameter logistic-curve function $y = k/(1+\exp(a+bx))$. A convenient logistic-function approximation for $p_f(D,\tau_0;1)$ was therefore given by Kastner & Bhatia (1998) as

$$p_f(D,\tau_0;1) \simeq \frac{a}{1+\exp\left(b\left(\log(\tau_0)-c\right)\right)} \quad (13)$$

where $a = 0.9999$, $b = 2.4105$ and $c = 0.39504$. This approximation holds within about $\pm 1\%$ for τ_0 up to 10. Thus $Q_1(\tau_0) \simeq; -\log(1+\exp(2.4105(\log\tau_0 - 0.39504)))$, as a convenient form for this SAC function when applicable. (FM gave an approximation $Q_1(\tau_0) \simeq -0.89\log\tau_0 + 0.41$, in the vicinity of $\tau_0 \simeq 100$. For this value of τ_0 the FM approximation gives $Q_1 = -1.37$, while the present approximation yields a larger value of $Q_1 = -1.69$. As mentioned above, however, the model function Q_1 holds strictly only for optical thicknesses below 10, losing validity above that value).

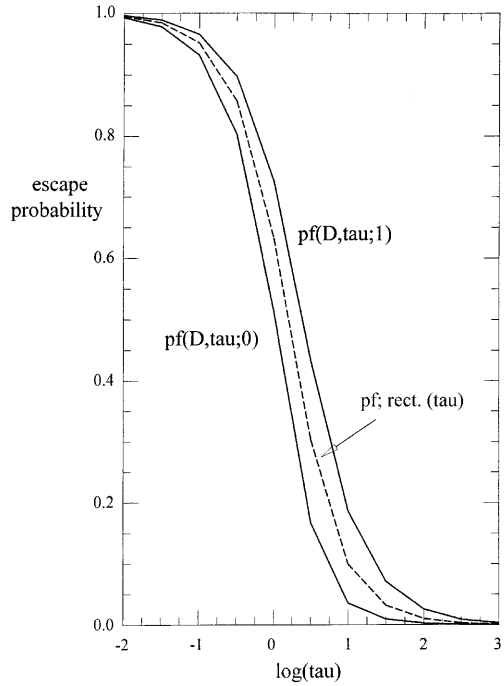

Fig. 1. Dependences of the uniform-source-function escape probability $p_f(D,\tau_0;1)$ and the localized-emission escape probability $p_f(D,\tau_0;0)$ on line-center optical depth τ_0. The dashed curve represents the escape probability $p_{f;rect.}(\tau)$ for a line of rectangular profile.

3.2. Candidate model SAC functions

The model function $Q_1(\tau_0)$ is a simple a priori physical model, being specified by only two parameters – one temperature (Doppler width) and one optical thickness τ_0 – so that it is one possible function to apply in the SAC method.

At the other extreme from the uniform source model $Q_1(\tau_0)$, for sources which can be characterized by localized emission behind a purely absorbing layer the appropriate escape probability is given by

$$p_f(D,\tau_0;0) = \frac{1}{\sqrt{\pi}\tau_0} \int_{-\infty}^{\infty} \exp\left[-x^2 - \tau_0\exp(-x^2)\right] dx \quad (14)$$

This escape probability is included in Fig. 1 for comparison with the uniform-source-function $p_f(D,\tau_0;1)$, and is seen to decrease more rapidly with increasing optical thickness as expected physically. A logistic function approximation of the form (13) is again available for this escape probability, with $a = 0.9911$, $b = 3.0206$ and $c = 0.001319$, so that the corresponding SAC model function is (using for convenience the Bartels' parameter $q = 0$ as subscript) $Q_0(\tau_0) \simeq -\log(1+\exp(3.0206(\log\tau_0 - 0.001319)))$.

The quantity $p_f(D,\tau_0;1)$ is the "upper limit" monodirectional escape probability analogous to Irons' fully-integrated escape factor Θ_{ul}, and the quantity $p_f(D,\tau_0;0)$ is the "lower limit" monodirectional escape probability analogous to Irons' escape factor Θ_{ll}. Their logarithms $Q_i(\tau_0)$ are plotted in Fig. 2; for Doppler line emission from (plane-parallel) source models, all physically realistic SAC functions $Q(\tau_0)$ should evidently

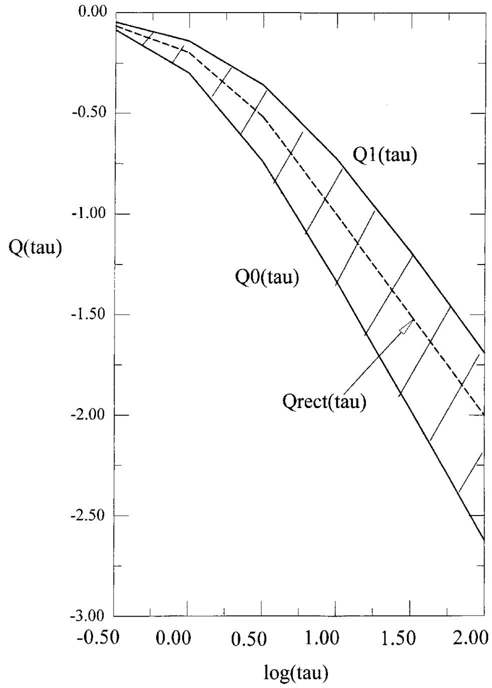

Fig. 2. τ_0-dependences of the SAC functions $Q_1(\tau_0) \equiv \log[p_f(D, \tau_0; 1)]$ and $Q_0(\tau_0) \equiv \log[p_f(D, \tau_0; 0)$. The shaded region between the curves is the allowed region for physically realizable static (plane-parallel) sources emitting Doppler-broadened emission lines. The dashed curve represents the corresponding SAC function for the rectangular profile; see Sect. 3.2. As emphasized throughout the discussion of Sect. 3, Figs. 1 and 2 lose validity above line-center optical thicknesses $\tau_0 < 10$, because the escape probability concept itself then loses validity.

lie in the shaded region between them, for $\log \tau_0 < 10$ (this region should apply reasonably well also to non-plane-parallel source geometries).

For higher optical thicknesses, as noted above, higher order scattered photons become important and the escape probability concept loses validity. Such SAC functions can therefore be generally represented at low optical thicknesses by the logistic-function representation, with the parameter set (b, c) to be solved for by, e.g., the least squares procedure employed by Muratorio et al. (1992).

FM used the rectangular line profile as a simplest model to treat by the SAC method. The corresponding escape probability is $p_{f;rect}(\tau) = (1 - \exp(-\tau))/\tau$. This escape probability is included also in Fig. 1 as the dashed curve, and it is seen that rather remarkably, it lies neatly between the extremes of the escape probabilities for uniform and sharply localized sources. Its corresponding SAC function $Q_{rect}(\tau)$, included as the dashed curve in Fig. 2, then similarly lies well within the physically realizable source region. $Q_{rect}(\tau)$ could therefore serve as a feasible and simple SAC function for real applications, having the added advantage that it involves no additional parameters to estimate. On the other hand, its particular location within the physical source region implies that it corresponds essentially to a particular source function (with a unique logistic-function set of parameters (b_{rect}, c_{rect})) and therefore its use would represent in principle an initial added constraint on any SAC solution.

For completeness, and to indicate the relation of the SAC method to other empirical diagnostic methods described in the literature, some other models are mentioned here which require more parameters for their specification but may be of potential use.

(a) Segmented (discrete) source models: More detailed/complex models have been employed for the description of static laboratory or astronomical emission line sources, which assume that the source is composed of two or more spatial segments with different (constant) source functions. These discrete-source-function models give explicit line profiles, which may then be integrated over as above to give corresponding photon escape probabilities and SAC functions $Q(\tau_0)$:

1. The West-Human model (West & Human 1976) assumes an extended homogeneous source behind an absorbing layer, i.e. two segments. It has been applied usefully to laboratory glow-discharge sources (cf. Payling et al. 1997). The (unnormalized) West-Human profile function is

$$\begin{aligned}\phi(\nu) &= \left[1 - \exp\left(-\tau_a \exp\left(-x_a^2\right)\right)\right] \\ &\quad \times \left[\exp\left(-\tau_b \exp\left(-x_b^2\right)\right)\right] \qquad (15)\end{aligned}$$

where τ_a and τ_b are the optical thicknesses respectively of the emitting/absorbing segment a and the purely absorbing layer b, and $x_a = (\nu - \nu_0)/\Delta\nu_a$, $x_b = (\nu - \nu_0)/\Delta\nu_b$ are dimensionless frequency variables corresponding to the two segments which have the Doppler halfwidths $\Delta\nu_a$, $\Delta\nu_b$.

2. The general model used by Li and co-workers (1994) to simulate various solar sources (prominences, flare loops, etc.) divides the source into two or more uniform-source-function segments, being essentially an extension of the West-Human two-segment model, so that the resulting line profile at a given wavelength λ is of the form (expression (18) of Li et al.):

$$\begin{aligned}I_\lambda &= \sum_{i=1}^{n} S_{\lambda_i}\left(1 - e^{-\tau_{\lambda_i}}\right) \exp\left[-\sum_{j=i+1}^{n+1} \tau_{\lambda_j}\right], \\ \tau_{\lambda_{n+1}} &= 0 \qquad (16)\end{aligned}$$

for n segments, in which emissions from segments successively further from the observer are successively more absorbed by the intervening segments.

Such models provide more realistic simulations of laboratory or astronomical sources, at least at low optical thicknesses such that line profiles are not self-reversed. On the other hand, they involve more parameters to estimate, requiring more extensive mathematical treatment.

(b) Non-segmented source models: Empirical continuous-source-function models intermediate between uniform and localized emission sources, applied to laboratory plasma discharges, have been described and reviewed by Fishman and co-

workers (Fishman, Il'in and Salakhov 1987) and by Karabourniotis (1990). These models, some yielding explicit line profiles, are often based on the original models of Bartels (1950a,b) or of Cowan & Dieke (1948), but have been extended in applicability by the more recent workers. They are applicable more generally to self-reversed line profiles, however at the cost usually of an assumption of local thermodynamic equilibrium (LTE) which is justified only for high-density plasma conditions.

4. Summary and discussion

The Friedjung-Muratorio Self-Absorption Curve (SAC) method is derived here from first principles in a more transparent manner and notation, and the SAC function $Q(\tau_0)$ is shown to be the logarithm of the photon escape probability appropriate to the chosen physical model of emission/absorption.

Because of this demonstrated importance of the escape probability concept in applications of the SAC method, it is discussed at some length to indicate the present state of development of the concept as well as its presently associated restrictions. Some concrete models now used in modelling laboratory and solar sources are described, of which the corresponding escape probability functions could in principle be used for the SAC method.

Not mentioned above was a point that because many of the lines or multiplet components in observed spectra have more general Voigt profiles rather than pure Doppler profiles, their escape probabilities and therefore their SAC curves will therefore be slightly different, and superpositions of multiplet curves may be possible only in an approximate sense.

FM and collaborators have applied the SAC method to rather complex astronomical source configurations, with encouraging results. At the same time, FM comment that "If one does not know exactly which detailed model can be used, radiative transfer... is complex, and the relevance of results of calculations, assuming it, is difficult to judge." It is suggested that just because of this complexity of the problem, the SAC method might profitably be applied to more well-defined laboratory emission sources, and to spectra which can be more accurately measured than astronomical spectra; the case of very precisely measured Fe II lines emitted from a glow-discharge source (Thorne et al. 1987; Kastner 1999) is an example. This would constitute a more definitive test of the SAC method and its assumptions.

Finally, Friedjung (1988) noted that "The theory of the form of the self absorption curves clearly needs to be refined. What is needed is theory which can be compared with the results of semi-empirical methods." The present interpretation of the SAC method is intended as a contribution toward this goal.

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