On the axis ratio of the stellar velocity ellipsoid in disks of spiral galaxies

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Abstract. The spatial distribution of stars in a disk of a galaxy can be described by a radial scale length and a vertical scale height. The ratio of these two scale parameters contains information on the axis ratio of the velocity ellipsoid, i.e. the ratio of the vertical to radial stellar velocity dispersions of the stars, at least at some fiducial distance from the center. The radial velocity dispersion correlates well with the amplitude of the rotation curve and with the disk integrated magnitude, as was found by Bottema (1993). These relations can be understood as the result of the stellar disk being (marginally) stable against local instabilities at all length scales. This is expressed by Toomre's well-known criterion, which relates the sheer in the rotation to a minimum value that the radial stellar velocity dispersion should have for stability for a given surface density. Via the Tully-Fisher (1977) relation, the velocity dispersion then becomes related to the integrated magnitude and hence to the scale length. The vertical velocity dispersion relates directly to the scale height through hydrostatic equilibrium. It can be shown that the ratio of the two length scales relates to the axis ratio of the velocity ellipsoid only through the Toomre parameter Q and in particular does not require a choice of the mass-to-light ratio or a distance scale.

We have applied this to the statistically complete sample of edge-on galaxies, for which de Grijs (1997) has performed surface photometry and has determined the length scales in the stellar light distribution.

Key words: galaxies: general – galaxies: kinematics and dynamics – galaxies: photometry

1. Introduction

It has been known for many decades that the distribution of the stellar velocities in the solar neighbourhood is far from isotropic. A longstanding problem in stellar dynamics (or in recent times more appropriately called galactic dynamics) has been the question of the shape and orientation of the velocity ellipsoid (i.e. the three-dimensional distribution of velocities) of stars in disks of spiral galaxies. The local ellipsoid has generated debate and

research for at least a century. There still is no consensus on the question of the orientation of the longest axis of the velocity ellipsoid (the "tilt" away from parallel to the plane) at small and moderate distances from the symmetry plane of the Galaxy (but see Cuddeford & Amendt 1991a,b), which is of vital importance for attempts to estimate the local surface density of the Galactic plane from vertical dynamics of stars. The radial versus tangential dispersion ratio is reasonably well understood as a result of the local sheer, which through the Oort constants governs the shape of the epicyclic stellar orbits (but see Cuddeford & Binney 1994 and Kuijken & Tremaine 1994 for higher order effects as a result of deviations from circular symmetry).

There are two general classes of models for the origin of the velocity dispersions of stars in galactic disks. The first, going back to Spitzer & Schwarzschild (1951), is scattering by irregularities in the gravitational field, later identified with the effects of Giant Molecular Clouds (GMCs). The second class of models can be traced back to the work of Barbanis & Woltjer (1967), who suggested transient spiral waves as the scattering agent; this model has been extended by Carlberg & Sellwood (1985). Recently, the possiblity of infall of satellite galaxies has been recognized as a third option (e.g. Velázquez & White, 1999).

In the solar neighbourhood the ratio of the radial and vertical velocity dispersion of the stars σ_z/σ_R is usually taken as roughly 0.5 to 0.6 (Wielen 1977; see also Gomez et al. 1990), although values on the order of 0.7 are also found in the literature (Woolley et al. 1977; Meusinger et al. 1991). The value of this ratio can be used to test predictions for the secular evolution in disks and perhaps distinguish between the general classes of models. Lacey (1984) and Villumsen (1985) have concluded that the Spitzer-Schwarzschild mechanism is not in agreement with observations: the predicted time dependence of the velocity dispersion of a group of stars as a function of age disagrees with the observed age – velocity dispersion relation (see also Wielen 1977), while it would not be possible for the axis ratio of the velocity ellipsoid σ_z/σ_R to be less than about 0.7 (but see Ida et al. 1993)

Jenkins & Binney (1990) argued that it is likely that the dynamical evolution in the directions in the plane and that perpendicular to it could have proceeded with both mechanisms contributing, but in different manners. Scattering by GMCs would then be responsible for the vertical velocity dispersion, while

scattering from spiral irregularities would produce the velocity dispersions in the plane. The latter would be the prime source of the secular evolution with the scattering by molecular clouds being a mechanism by which some of the energy in random motions in the plane is converted into vertical random motions, hence determining the thickness of galactic disks. The effects of a possible slow, but significant accretion of gas onto the disks over their lifetime has been studied by Jenkins (1992), who pointed out strong effects on the time dependence of the vertical velocity dispersions, in particular giving rise to enhanced velocities for the old stars.

The only other galaxy in which a direct measurement of the velocity ellipsoid has been reported, is NGC488 (Gerssen et al. 1997). NGC488 is a moderately inclined galaxy, which enables these authors to solve for the dispersions from a comparison of measurements along the major and minor axes. NGC488 is a giant Sb galaxy with a photometric scale length of about 6 kpc (in the B-band) and an amplitude of the rotation curve of about 330 km s^{-1}. The axis ratio σ_z/σ_R is 0.70 ± 0.19; this value, which is probably larger than that for the Galaxy, suggests that the spiral irregularities mechanism should be relatively less important, in agreement with the optical morphology.

The light distribution in galactic disks has in the radial direction an exponential behaviour (Freeman 1970), characterised by a scale length h. In the vertical direction –at least away from the central layer of young stars and dust that is obvious in edge-on galaxies– it turns out that the light distribution can also be characterised by an exponential scale height h_z, which is independent of galactocentric distance (van der Kruit & Searle 1981, but see de Grijs & Peletier 1997). It then is usually assumed that the three-dimensional light distribution traces the distribution of mass; this seems justifiable since the light measured is that of the old disk population, which contains most of the stellar disk mass and dominates the light away from the plane. On general grounds, these two typical length scales are expected to be independent, the radial one resulting from the distribution of angular momentum in the protogalaxy (e.g. van der Kruit 1987; Dalcanton et al. 1997) or that resulting from the merging of pre-galactic units in the galaxy's early stages, while the length scale in the z-direction would result from the subsequent, and much slower, disk heating and the consequent thickening of the disk. It is not *a priori* clear, therefore, that the two scale lengths should correlate. Yet, they do bear a relation to the ratio of the velocity dispersions of the stars in the old disk population. The vertical one follows directly from hydrostatic equilibrium. In the radial direction it is somewhat indirect; a relation between the radial scale length and the corresponding velocity dispersion comes about through conditions of local stability (e.g. Bottema 1993).

In a recent study, de Grijs (1997, 1998; see also de Grijs & van der Kruit 1996) has determined the two scale parameters in a statistically complete sample of edge-on galaxies and found the ratio of the two (h/h_z) to increase with later morphological type. In this paper we will examine this dataset in detail in order to investigate whether it can be used to derive information on the axis ratios of the velocity ellipsoid and help make progress in resolving the general issues described above.

2. Background

In an extensive study of stellar kinematics of spiral galaxies, Bottema (1993) presented measurements of the stellar velocity dispersions in the disks of twelve spiral galaxies. This first reasonably sized sample represented a fair range of morphological types and luminosities, although it was not a complete sample in a statistical sense. In each galaxy he determined a fiducial value for the velocity dispersion, namely at one photometric (B-band) scale length. He then found that this fiducial velocity dispersion correlated well with the absolute disk luminosity as well as with the maximum rotation velocity of the galaxy. His sample contained both highly inclined galaxies (where the velocities in the plane are in the line of sight) and close to face-on systems (where one measures the vertical velocity dispersion); when he forced the relations for the two classes of galaxies to coincide he found that a similar ratio between radial and vertical velocity dispersion as applicable for the solar neighbourhood was needed.

Bottema's empirical relation for velocity dispersion versus rotation velocity is

$$\sigma_{R,h} = 0.29 \, V_{rot}, \tag{1}$$

whereas for velocity dispersion versus disk luminosity it reads (in the form of absolute magnitude)

$$\sigma_{R,h}(\text{km s}^{-1}) = -17 \times M_B - 279. \tag{2}$$

These relations can –for any galaxy for which the photometry is available or for which the rotation curve is known– be used to estimate the radial stellar velocity dispersion of the old disk stars at one photometric scale length from the center. Doing this for the de Grijs sample of edge-on galaxies and estimating the vertical velocity dispersion from the vertical scale height, one can in principle determine the axis ratio of the velocity ellipsoid for this entire sample. It would appear that this is a rather uncertain procedure, since one will have to assume a mass-to-light ratio (M/L) in order to calculate the vertical velocity dispersion from the photometric parameters. We will show, however, that M/L does not enter explicitly in the formula for the ratio of velocity dispersions.

We will list our assumptions:

• The surface density of the disk has an exponential form as a function of galactocentric distance:

$$\Sigma(R) = \Sigma(0) \, e^{-R/h}. \tag{3}$$

• The vertical distribution of density can be approximated by that of the isothermal sheet (van der Kruit & Searle 1981), but we will use instead the subsequently suggested modification (van der Kruit 1988)

$$\rho(R,z) = \rho(R,0) \, \text{sech}(z/h_z). \tag{4}$$

A detailed investigation of the sample (de Grijs et al. 1997) shows indeed that the vertical light profiles are much closer to exponential than to the isothermal solution, although the mass density distribution most likely is less peaked than that of the

light, since young populations with low velocity dispersions add significantly to the luminosity but little to the mass. Then the vertical velocity dispersion σ_z can be calculated from

$$\sigma_z^2 = 1.7051\pi G \Sigma(R) h_z. \tag{5}$$

The usual parameter $z_\circ$ used in the notation for the isothermal disk (and in de Grijs 1998) is $z_\circ = 2h_z$. It is important to note that this formula assumes that the old stellar disk is self-gravitating. Although this can be made acceptable for galaxies like our own at positions of a few radial scale lengths from the center (see van der Kruit & Searle 1981), it is improbable in late-type galaxies, which have significant amounts of gas in the disks, and we will need to allow for this.

• The mass-to-light ratio M/L is constant as a function of radius. Support for this comes from the observation by van der Kruit & Freeman (1986) and Bottema (1993) that the vertical velocity dispersion in face-on spiral galaxies falls off with a scale length about twice that of the surface brightness (but note that Gerssen et al. 1997, could not confirm this for NGC488), combined with the observed constant thickness of disks with galactocentric radius.

• We are not making any assumptions on the functional form of the dependence of the radial velocity dispersion or the axis ratio of the velocity ellipsoid. The observed radial stellar velocity dispersions in Bottema's sample are consistent with a drop-off $\propto \exp(-R/2h)$, in which case this axis ratio would be constant with galactocentric distance. However, over the range considered the data can be fitted also with a radial dependence for the radial velocity dispersion in which the parameter Q for local stability against axisymmetric modes (Toomre 1964) is constant with radius ($\propto R\exp(-R/h)$; see van der Kruit & Freeman 1986). The definition of Toomre's (1964) parameter Q for local stability against axisymmetric modes is

$$Q = \frac{\sigma_R \kappa}{3.36 G \Sigma}. \tag{6}$$

Disks are stabilised at small scales through the Jeans criterion by random motions (up to the radius of the Jeans mass) and for larger scales by differential rotation. Toomre's condition states that the minimum scale for stability by differential rotation should be no larger than the Jeans radius.

• We assume that spiral galaxies have flat rotation curves with an amplitude $V_{\rm rot}$ over all but their very central extent. This assumption implies that we may write the epicyclic frequency κ as

$$\kappa = 2\sqrt{B(B-A)} = \sqrt{2}\frac{V_{\rm rot}}{R}, \tag{7}$$

where A and B are the Oort constants.

First we will look into the background of the Bottema relations (1) and (2) (see also van der Kruit 1990; Bottema 1993, 1997).

Evaluating Toomre's Q at $R = 1h$ and using the expression for the epicyclic frequency above, we find

$$\sigma_{\rm R,h} = \frac{3.36\,G}{\sqrt{2}}Q\frac{\Sigma(h)h}{V_{\rm rot}}. \tag{8}$$

Using $\Sigma(0) = (M/L)\mu_\circ$ and the total disk luminosity from $L_{\rm d} = 2\pi\mu_\circ h^2$ we get

$$\sigma_{\rm R,h} = \frac{1.68 G}{{\rm e}\sqrt{\pi}}Q\left(\frac{M}{L}\right)\frac{\mu_\circ^{1/2}L_{\rm d}^{1/2}}{V_{\rm rot}}. \tag{9}$$

Neither in the sample of galaxies that Bottema used to define his relations, nor in our sample of edge-on systems do we have galaxies with unusually low surface brightness. It seems therefore justified to assume that for the galaxies considered we have a reasonably constant central surface brightness (Freeman 1970; van der Kruit 1987)

$$\mu_\circ \approx 21.6 \ \mu_B = 142\,{\rm L}_\odot\,{\rm pc}^{-2}, \tag{10}$$

where μ_B stands for B-magnitudes arcsec^{-2}. So, if $\mu_\circ$, Q and (M/L) are constant between galaxies, we see that the fiducial velocity dispersion depends only on the disk luminosity and the rotation velocity.

Bottema's relation (1) can then be reconciled with Eq. (9), if we have

$$L_{\rm d} \propto V_{\rm rot}^4. \tag{11}$$

This is approximately the Tully-Fisher relation (Tully & Fisher 1977); not precisely, since we use the disk luminosity and not that of the galaxy as a whole (however, for late-type galaxies this would be a minor difference)[1].

So, we see that Bottema's relation (1) follows directly from Toomre's stability criterion in exponential disks with flat rotation curves as long as Eq. (11) holds. The proportionality constant in Eq. (11) can be fixed using the parameters for the Milky Way Galaxy and for NGC 891 as given in van der Kruit (1990). These two galaxies have $L_{\rm d} \sim 1.9 \times 10^{10}\,{\rm L}_\odot$ and $V_{\rm rot} \sim 220\,{\rm km\,s}^{-1}$[2]. This gives

$$L_{\rm d}\,({\rm L}_\odot) = 8.11\,V_{\rm rot}^4\,({\rm km\,s}^{-1}). \tag{12}$$

and using also Eq. (10) we get

$$\sigma_{\rm R,h} = 5.08 \times 10^{-2}Q\left(\frac{M}{L}\right)V_{\rm rot}. \tag{13}$$

From this we find with Bottema's relation (1), that $Q(M/L)_B \approx 5.7$. In a somewhat different, but comparable manner, Bottema (1993) has also concluded that this product is of order 5.

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[1] Eqs. (1) and (2) together would imply an exponential Tully-Fisher relation rather than a power law. The problem is that Bottema chose to fit a linear relation to his data, which is completely justified in view of his error bars. But Eqs. (11) and (9) would imply that he should have fitted a curve in which the velocity dispersion is proportional to $L_{\rm d}^{1/4}$ (see van der Kruit 1990, p. 199). Performing such a fit to Bottema's data gives a curve that is only marginally different from a straight line over his range of absolute magnitudes (only a few km s^{-1}). So, Eq. (2) should only be seen as an empirical fit of the data to a straight line, although these data are equally consistent with the dependence following from Eqs. (9) and (11).

[2] The distance for NGC 891 –as are all distances in this paper– is based on a Hubble constant of 75 km s^{-1} Mpc^{-1}.

We now turn to the vertical velocity dispersion. Evaluating the equation for hydrostatic equilibrium (5) at galactocentric distance $R = 1h$ we find

$$\sigma_{z,h} = \{5.36 \, G\Sigma(h)h_z\}^{1/2}, \tag{14}$$

and can thus calculate the vertical velocity dispersion from

$$\sigma_{z,h} = \left\{\frac{5.36}{e} \, G\left(\frac{M}{L}\right)\mu_\circ h_z\right\}^{1/2}. \tag{15}$$

Finally we examine the ratio of the two velocity dispersions. If we eliminate $\Sigma(h)$ between Eqs. (8) and (14) we obtain

$$\sigma_{R,h} = 0.444 \, Q \frac{\sigma_{z,h}^2}{V_{rot}} \frac{h}{h_z}, \tag{16}$$

and with Eq. (1)

$$\left(\frac{\sigma_z}{\sigma_R}\right)_h^2 = \frac{7.77}{Q}\frac{h_z}{h}. \tag{17}$$

Note that due to the elimination of the surface density also the mass-to-light ratio has dropped out of this equation and the result is independent of any assumption on M/L. Eq. (17) translates the ratio of the two length scales to that of the corresponding velocity dispersions and the underlying physics can be summarized as follows. In the vertical direction the length scale and the velocity dispersion relate through dynamical equilibrium. In the radial direction the velocity dispersion is related to the epicyclic frequency through the local stability condition, which is proportional to the rotation velocity. The "Tully-Fisher relation" then relates this to the integrated magnitude and hence to the size and length scale of the disk.

One should be careful in the use of Eq. (17), since in practice photometric scale lengths are wavelength dependent and its derivation –and therefore the numerical constant– is valid only at one exponential *surface density* scale length. The purpose of presenting it here is only to show that, if the two velocity dispersions are derived in a consistent manner from Eqs. (8) and (14) –or alternatively Eqs. (9) and (15)–, the assumption used for M/L drops out in the resulting ratio.

3. Application to the de Grijs sample

The sample of edge-on disk galaxies of de Grijs (1998) contains 46 systems for which the structural parameters of the disks have been determined (including a bulge/disk separation in the analysis). From this sample we take those for which rotation velocities have been derived in a uniform manner (Mathewson et al. 1992; data collected in de Grijs 1998, Table 4) as well as those for which the Galactic foreground extinction in the *B*-band is less than 0.25 magnitudes. For this remaining sample of 36 galaxies we perform the following calculations:

• From the total magnitudes in de Grijs (1998, Table 6) we obtain the integrated *B*- and *I*-magnitudes of the disk.
• Using the radial scale length as measured in the *I*-band we calculate the central (face-on) surface brightness of the disk

from its *I*-band integrated luminosity. So, we do not use Eq. (10) for a constant central surface brightness.
• Then we use one or both of the two Bottema relations (1) and (2) to estimate the radial velocity dispersion at one photometric scale length (by definition in the *B*-band). Where we can do this with both relations, the ratio between the two estimates is 1.11 ± 0.19. This is not trivial, as the rotation velocities and disk luminosities are determined completely independently (and by different workers) and only the one using the absolute magnitude needs an assumption for the distance scale.
• Then using Eq. (15), we estimate the vertical velocity dispersion at one photometric scale length in the *I*-band. For this we need a value for the mass-to-light ratio and we will discuss this first.

We found, that through Eq. (13) Bottema's relation (1) provides a value for $Q(M/L)$ of about 5.7. So we make a choice for Q rather than for M/L. It has become customary to assume values of Q of order 2, mainly based on the numerical simulations of Sellwood & Carlberg (1984), who find their disks to settle with $Q \sim 1.7$ at all radii. In principle we can use the observed properties of the Galaxy to fix Q from Eq. (17). We have $(\sigma_z/\sigma_R)^2 \sim 0.5$ (in the solar neighbourhood, but assume for the sake of the argument also at $R = 1h$) and $h_z/h \sim 0.1$ (see Sackett 1997 for a recent review), so that indeed $Q \sim 1.7$. We will make the general assumption that $Q = 2$, in agreement with the considerations above; then $(M/L)_B = 2.8$.

The rotation velocity version of Bottema's empirical relations (Eq. (1)) can provide further support for the choice of Q, along the lines of the discussion in van der Kruit & Freeman (1986). In the first place we recall the condition for the prevention of swing amplification in disks (Toomre 1981), as reformulated by Sellwood (1983)

$$X = \frac{R\kappa^2}{2\pi m G\Sigma} > 3, \tag{18}$$

where m is the number of spiral arms. For a flat rotation curve this can be rewritten as

$$\frac{QV_{rot}}{\sigma_R} > 3.97m. \tag{19}$$

With Eq. (1) this becomes $Q > 1.15m$. Considering that the coefficient in Eq. (1) has an uncertainty of order 15%, this tells us that we have to assume Q at least of order 2 to prevent strong barlike ($m=2$) disturbances in the disk.

A similar argument can be made using the global stability criterion of Efstathiou et al. (1982). This criterion states that for a galaxy with a flat rotation curve and an exponential disk, global stability requires a dark halo and

$$Y = V_{rot}\left(\frac{h}{GM_{disk}}\right)^{1/2} \gtrsim 1.1. \tag{20}$$

Here M_{disk} is the total mass of the disk. This can be rewritten as

$$Y = 0.615\left[\frac{QRV_{rot}}{h\sigma_R}\right]^{1/2}\exp\left(-\frac{R}{2h}\right) \gtrsim 1.1, \tag{21}$$
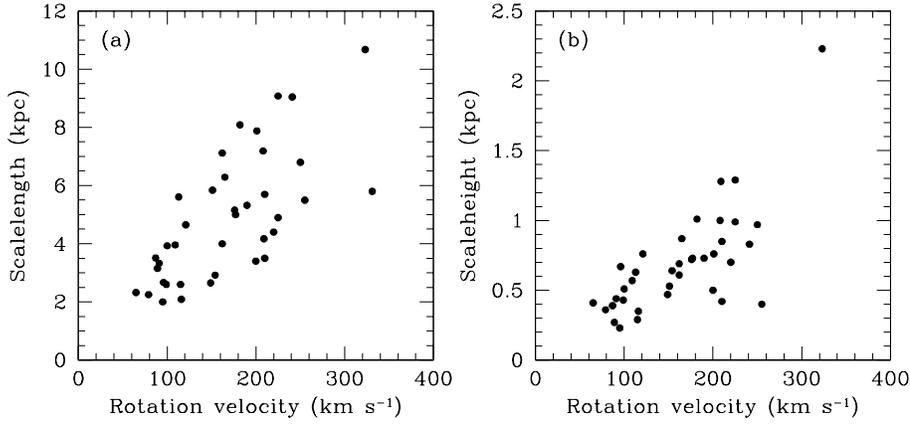
Fig. 1a and b. The scale lengths **a** and scale heights **b** of the galaxies in our sample as a function of their rotation velocities. The r.m.s. errors are of order $10\,\mathrm{km\,s^{-1}}$ in the rotation velocities (Mathewson et al. 1992), 5% in the radial scale length and 9% in the vertical scale height (de Grijs 1998).

and, when evaluated at $R = 1h$, yields with Eq. (1) $0.69\sqrt{Q} \gtrsim 1.1$, and therefore also implies that Q should be at least about 2. Efstathiou et al. have also come to this conclusion for our Galaxy, with the use of local parameters for the solar neighbourhood.

Having adopted a value for Q and through this a value for $(M/L)_B$, we will have to convert it to $(M/L)_I$. For this we need a $(B-I)$ colour for the disks. From the fits of de Grijs (1998) we find that the total *disk* magnitudes show a rather large variation in colour; for the sample used here $(B-I)$ has a mean value of 1.9, but the r.m.s. scatter is 0.8 magnitudes. In his discussion, de Grijs (1998) suspects a systematic effect of the internal dust in the disks (particularly on the *B*-magnitudes, which is another reason for us to use the V_{rot}-version of the Bottema relation (Eq. (1)) in our derivation in the previous section). Instead we turn to the discussion of de Jong (1996b), who compares his surface photometry of less inclined spirals to star formation models. From his Table 3, we infer that for single burst models with solar metallicity and ages of $12\,\mathrm{Gyr}\,(M/L)_B = 2(M/L)_I$. So we will use an $(M/L)_I$ of 1.4.

There is a further refinement required. In order to take into account the fact that in late-type galaxies the gas contributes significantly to the gravitational force, we have to correct for a galaxy's gas content as a function of Hubble type. In the following, we will discuss the observational data regarding the HI and the H_2 separately.

For 25 of de Grijs' sample galaxies HI observations are available, so that we can estimate the gas-to-total disk mass. For this we apply a correction of a factor 4/3 to the HI in order to take account of helium and use de Grijs' (1998) *I*-band photometry and our adopted M/L_B ratio of 2.8 (see below) to estimate the total disk mass. As a function of Hubble type we then find

Type	gas-to-total disk mass	n
Sb	0.31 ± 0.17	3
Sbc	0.36 ± 0.19	5
Sc	0.53 ± 0.09	5
Scd	0.49 ± 0.16	9
Sd	0.52 ± 0.10	3

We find no dependence on rotation velocity:

| V_{rot} (km s^{-1})	gas-to-total disk mass	n
80–130	0.51 ± 0.09	10
130–180	0.45 ± 0.17	8
180–230	0.51 ± 0.09	7

So, the HI mass is about half the stellar mass in disks of Sb's and about similar to that in Sc's and Sd's. But there is no dependence on rotation velocity. But this is not what we need; we should use surface densities rather than disk masses. Now, the HI is usually more extended than the stars and has a shallower radial profile. So the ratios in the tables above are definite upper limits. In order to take into account the effect that in late-type systems the gas contributes significantly to the gravitational force we have "added" for types Scd and Sd a similar amount of gas as in stars and half of that for Sc's.

The distribution of H_2 in spiral galaxies is a more complex matter; it is often centrally peaked, although some Sb galaxies exhibit central holes (for a recent review, see Kenney 1997). The molecular fraction of the gas appears to be lower in low-mass and late-type galaxies, assuming that the conversion factor from CO to molecular hydrogen is universal[3]. Since our sample galaxies are generally low-mass, later-type systems, we believe that the corrections for molecular gas are small, and therefore contribute little to the correction for the presence of gas.

We added (a) the galaxies from van der Kruit & Searle (1982) to the sample, (b) our Galaxy using the Lewis & Freeman (1989) velocity dispersion and the structural parameters in van der Kruit (1990), and (c) the observational results for NGC488 from Gerssen et al. (1997). We leave the few early type (S0 and Sa) galaxies out of the discussion, because the component seperation in the surface brightness distributions is troublesome and some of our assumptions (in particular the self-gravitating nature of the disks) are probably seriously wrong.

In order to be able to trace the origin of our results, we first show in Fig. 1 the radial and vertical scale lengths of the sample as a function of the rotation velocity. Both increase with V_{rot}, which would be expected intuitively. The main result is presented in Figs. 2 and 3. From Fig. 2 we see that the vertical

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[3] There is even doubt concerning the constancy of this conversion factor *within* our Galaxy (Sodroski et al. 1995).
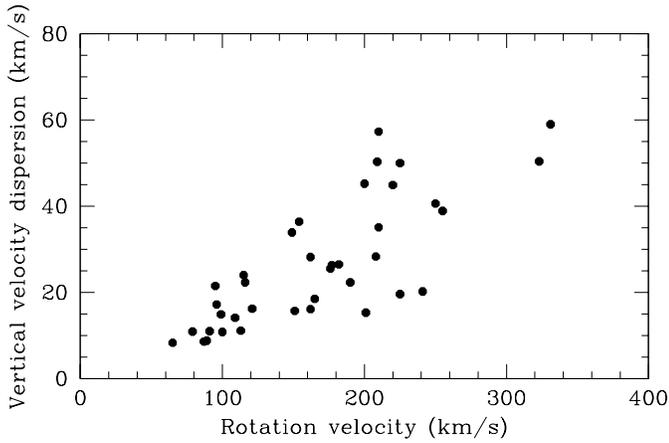
Fig. 2. The calculated vertical velocity dispersions at one exponential scale length from the center as a function of the rotation velocities

Fig. 3. The ratio of the calculated vertical to radial velocity dispersions at one exponential scale length from the center as a function of the rotation velocities

velocity dispersions, that have been derived from hydrostatic equilibrium, increase with the rotation speed (the radial velocity dispersions do the same automatically as a result of the use of the Bottema relations). For the slowest rotation speeds the predicted vertical velocity dispersion is on the order of 10–$20\,\mathrm{km\,s^{-1}}$, which is close to that observed in the neutral hydrogen in face-on galaxies (van der Kruit & Shostak 1984).

The distribution of the axis ratio of the velocity ellipsoid with morphological type is as follows:

Type	$\sigma_{z,h}/\sigma_{R,h}$	n
Sb	0.71 ± 0.14	11
Sbc	0.69 ± 0.16	7
Sc	0.49 ± 0.17	6
Scd	0.70 ± 0.20	11
Sd	0.63 ± 0.22	5

Not much of a trend is seen here. It is in order to comment here briefly on the effects of our corrections for the gas to obtain vertical velocity dispersions. From our discussion above we conclude that there would be no systematic effect introduced as a function of rotation velocity. Furthermore, taking away our correction altogether reduces the values for the average axis ratio in the table just given to about 0.55 for Scd and Sd galaxies. Even in this unrealistic case of not allowing for the presence of the gas, we believe the trend to be hardly significant in view of the uncertainties. Since some correction for the gas mass as a function of morphological type must be made, we cannot claim that we find any evidence for a change in the velocity anisotropy with Hubble type.

Fig. 3 shows the axis ratio of the velocity ellipsoid of all the galaxies versus their rotation velocity. In view of the fact that the two dispersions that go into this ratio are determined from different observational data ($\sigma_{R,h}$ from integral properties such as total luminosity and amplitude of the rotation curve; $\sigma_{z,h}$ from photometric scale parameters and surface brightness) and that we have made rather simplifying assumptions, the scatter is remarkably small. No systematic trends are visible (and
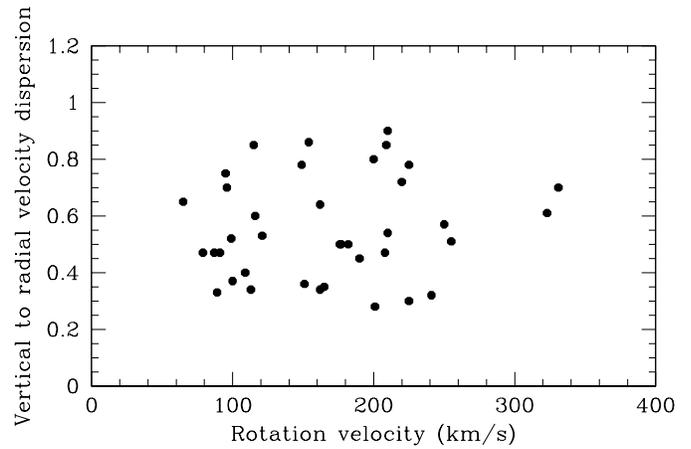
would probably not be significant!) in the data. The points closest to unity in the dispersion ratio generally have low rotation velocities and inferred velocity dispersions. One of these ($V_{\rm rot}$ = $95\,\mathrm{km\,s^{-1}}$, $\sigma_{z,h}/\sigma_{R,h}$ = 0.75) is NGC5023. Bottema et al. (1986) have shown that the stars and the gas in this galaxy are effectively coexistent; the radial *and* vertical distributions are very similar. This would imply that the velocity dispersions of the gas and the stars are the same. The vertical velocity dispersion found here is about $20\,\mathrm{km\,s^{-1}}$, which is significantly higher than that observed in larger spirals. It would be of interest to measure the H I velocity dispersion in this galaxy. Since the HI would be expected to have an isotropic velocity distribution from collisions between clouds, the vertical dispersion should be equal to that in the line of sight in edge-on galaxies.

4. Discussion

In this section we will critically discuss the uncertainties in our approach.

The linearity of the magnitude version of the Bottema relation.
We have discussed above that the power-law nature of the Tully-Fisher relation (Eq. (11)) would imply a nonlinear form of the magnitude version of Bottema's relation (Eq. (2)). We have used it as an empirical relation to help (together with Eq. (1)) to estimate the radial velocity dispersions from the observed photometry. One may argue that it is internally consistent to use instead of Eq. (2) a fit of the form

$$\sigma_{R,h} \propto L_{d}^{1/4}. \tag{22}$$

This has only a noticable effect on galaxies with faint absolute disk magnitudes. We have repeated our analysis using such a fit and find no change in our results. To be more definite we repeat the table of the average axis ratio as a function of morphological type that we then obtain.

Type	$\sigma_{z,h}/\sigma_{R,h}$	n
Sb	0.67 ± 0.17	11
Sbc	0.64 ± 0.13	7
Sc	0.47 ± 0.17	6
Scd	0.60 ± 0.11	11
Sd	0.60 ± 0.20	5

Choice of Q. We have adopted a value for Q of 2.0, and this enters directly in our results in the calculation of the vertical velocity dispersion through the value of M/L that follows from this choice. Had we adopted a value of 1.0 for Q, M/L would have been a factor 2 higher and the value for $\sigma_{z,h}$ a factor $\sqrt{2}$ –see Eq. (15)–. Bottema (1993, his Fig. 11) has shown that his observations of the stellar kinematics do not show any evidence for systematic variations in Q among galaxies.

We have used Eq. (19) to argue that Q is likely of order 2 in order to prevent strong barlike (m=2) disturbances. It does not necessarily follow from this that galaxies with more spiral arms should have higher values of Q or that Q should be significantly lower in galaxies with very strong two-armed structure. Spiral structure may arise in a variety of ways; we only argue that disks do not have grossly distorted m=2 shapes and that therefore swing amplification is apparently not operating.

We have assumed that the stellar disk is self-gravitating and ignored the influence of the gas in the evaluation of Q. At one scale length this is probably justified (it is only a small effect in solar neighbourhood), even for late-type disks.

Non-exponential nature of the disks. Often the disk in actual galaxies can be fitted to an exponential only over a limited radial extent. In that case our description is unlikely to hold. However, in our sample the fits can be made reasonably well at one scale length from the center and we believe this not to be a problem.

Vertical structure of the disks. Our results depend on the adoption of a particular form of the vertical mass distribution (namely the sech(z)-form) of Eq. (4). This enters our results through the value of the numerical constant 1.7051 in Eq. (5), and in Eq. (15) it enters into the value for $\sigma_{z,h}$ as its square root. Had we assumed the isothermal distribution, then the constant would have been 2.0, while it would have been 1.5 for the exponential distribution. This would have given us values for $\sigma_{z,h}$ which are only 6 to 8% higher or lower. As we have shown in de Grijs et al. (1997), for our sample galaxies the vertical *luminosity* distributions in these disk-dominated galaxies are slightly rounder than or consistent with the exponential model. However, the vertical *mass* distribution is probably less sharply peaked, and thus expected to be more closely approximated by the sech(z) model.

Non-constancy of central (face-on) surface brightness. We have assumed in Sect. 2 that for all galaxies the central surface brightness is constant. This is certainly unjustified for so-called "low surface brightness galaxies"; however, our galaxies have

brighter surface brightnesses than galaxies that are usually considered to be of this class. But even for galaxies as in our sample it remains true that the (face-on) central surface brightness is in general somewhat lower for smaller systems (van der Kruit 1987; de Jong 1996a). From de Jong's bivariate distribution functions, it can be seen that late-type, low absolute magnitude spirals may have a central surface brightness (in *B*) that is up to 1.0 magnitude fainter. Note, however, that for our derived velocity dispersons we have used the *actually observed* surface brightness for each galaxy.

Effects of colour variations on the mass-to-light ratio. There is a fairly large variation in the colours of the disks in the sample. De Grijs (1998) has argued that this is the result of internal dust extinction. However, we have used the *I*-band data; de Grijs' Fig. 11 shows that the variation is much less in $(I - K)$ than in colours involving the *B*-band.

The colours of the disks do not correlate with morphological type (de Grijs 1998); although such a correlation has been seen in more face-on galaxies (de Jong 1996b), we believe that the fitting procedure has ignored most of the young population and dust absorption near the plane, and that contributions to the M/L scatter as a result of young populations has mostly been avoided; and that the *I*-band scale lengths determined away from the galactic planes are fairly representative of the stellar mass distributions (de Grijs 1998).

The colour of the fitted disks in our sample has $(I - K)$ in the range ~ 2 to 4. The latter is red, even for an old population and may well be caused by excessive internal extinction, but we see no strong evidence for a substantial systematic correction in our velocity dispersions from this.

Effects of metallicity on the mass-to-light ratio. De Jong (1996b) has drawn attention to the non-negligible effects of metallicity on the mass-to-light ratio. From his compilation of models, in particular his W94 (Worthey 1994) models with ages of 12 Gyr, we estimate that the effect in the *I*-band amounts to 10 to 20% over the range of relevant metallicities. The effect on the derived velocity dispersions is the square root of this.

Effects of radial colour variations. Since we use an empirical relation to derive the radial velocity dispersion at one *B*-scale length from the center, we have to consider the effect of using the *I*-band. We have used the latter as the proper scale length to use for the mass density distribution. The correct one to use here would be the one measured in the *K*-band, which is 1.15 ± 0.19 times smaller for this sample. The scale length in the *B*-band is 1.64 ± 0.41 times longer than in the *K*-band (values quoted here from de Grijs 1998). We deduce from this that we may have underestimated the scale length to use by a factor 1.43 and systematically overestimated the vertical velocity dispersion by about 20%.

The effect of radial metallicity variations on the scale length is probably very small; these gradients in the older stellar pop-

ulations are in any case expected to be significantly less than in the interstellair medium. This is so, because in models for galactic chemical evolution the mean stellar metallicity of the stars approximates the (effective) yield, while that in the gas grows to much large values in most models (van der Kruit 1990, p. 322).

Non-flatness of rotation curves. This may be an effect for small, late-type galaxies that have slowly rising rotation curves. It enters however in our analysis only in the derivation of the Bottema relations and this holds empirically to rather small rotation velocities ($\sim 100\,\mathrm{km\,s^{-1}}$).

The slope of the Tully-Fisher relation. Although we use a relation between the luminosity of the disk alone and the rotation velocity, it remains true that our "slope" of V_{rot}^4 is steeper than the usually derived slopes of Tully-Fisher relations, which would indicate V_{rot}^3 (e.g. Giovanelli et al.'s 1997 "template relation" yields an exponent of 3.07 ± 0.05). If this slope were to be put into Eq. (9), Eq. (1) would have $\sigma_{\mathrm{R,h}} \propto \sqrt{V_{\mathrm{rot}}}$, which is very significantly in disagreement with Bottema's observations. The same holds for the other Bottema relation, Eq. (2).

The analysis of the present sample (see de Grijs & Peletier 1999) has resulted in slopes in the Tully-Fisher relation of 3.20 $\pm$ 0.07 in the *I*-band and 3.24 $\pm$ 0.21 in the *K*-band. On the other hand, Verheijen (1997), in his extensive study of about 40 galaxies in the Ursa Major cluster, finds a slope of 4.1 $\pm$ 0.2 in the *K*-band.

Effects of the gas on the value of Q. In our calculations we have already made crude allowance for the effects of the gas on the gravitational field. But in our derivations we have not taken into account the effect of the H I on the effective velocity dispersion to be used in the evaluation of the Q-parameter. The effect of the H I is a decrease of the effective velocity dispersion and therefore in Q. This means that the assumed value should in reality be decreased on average, but beyond this numerical effect, it does not affect our results.

The effects just discussed can produce errors in the estimated velocity dispersions of the order of 10 to 20% each. The final result of the dispersion ratios in Fig. 3 may therefore be wrong by a few tenths, which is comparable to the scatter in that figure. However, we have no cause to suspect that we have introduced serious systematic effects that would be strong functions of the rotation velocity or the morphological type and the lack of correlation of the axis ratio with these properties is unlikely to be an artifact of our analysis.

We conclude that it is in principle possible to infer information on the axis ratio of the velocity ellipsoid from a sample of edge-on galaxies for which both the radial scale length as well as the vertical scale height have been measured. The result, however shows much scatter, most of which is a result of the necessary assumptions. There is one significant improvement that can be made and that is the direct observation of the stellar velocity dispersion in these disks. That this is feasible in practice for edge-on systems has been shown by Bottema et al. (1987, 1991). The observed velocity profiles can be corrected for the line-of-sight effects, giving the tangential velocity dispersion, which through the observed shape of the rotation curve can be turned into the radial velocity dispersion. Although a time consuming programme, we believe that it is worth doing for two reasons: (1) It will set both versions of the Bottema relation on a firmer footing. (2) The uncertainties in the analysis above can likely be significantly diminished by direct measurement of the radial velocity dispersion rather than having to infer it from the rotation velocity or the disk absolute magnitude.

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