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The population, magnitudes, and sizes of Jupiter family comets

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Abstract. We analyze the sample of measured nuclear magnitudes of the observed Jupiter family (JF) comets (taken as those with orbital periods $P < 20$ years and Tisserand parameters $T > 2$). We find a tendency of the measured nuclear magnitudes to be fainter as JF comets are observed with CCD detectors attached to medium- and large-size telescopes (e.g. Spacewatch Telescope). However, a few JF comets observed very far from the Sun (4–7 AU) show a wide dispersion of their derived absolute nuclear magnitudes which suggests that either these JF comets keep active all along the orbit, so the reported unusually bright distant magnitudes were strongly contaminated by a coma, or some of the measured "nuclear magnitudes" were grossly overestimated (i.e. their brightness underestimated).

The cumulative mass distribution of JF comets is found to follow a power-law of index $s = -0.88 \pm 0.08$, suggesting a distribution significantly steeper than that for both small main-belt asteroids and near-Earth asteroids. The cumulative mass distribution of JF comets with $q < 2$ AU tends to flatten for absolute (visual) nuclear magnitudes $H_N \gtrsim 16$, which is probably due to incompleteness of discovery of fainter comets and/or a real scarcity of small comets due, perhaps, to much shorter physical lifetimes. In particular, no JF comets fainter than $H_N \sim 19.5$ are found in the sample, suggesting that the critical size for a comet to be still active may be of about 0.4 km radius for an assumed geometric albedo of 0.04. Possibly, smaller comet nuclei disintegrate very quickly into meteor streams. Most absolute nuclear magnitudes are found in the range 15–18, corresponding to nuclear radii in the range 0.8–3.3 km (for the same geometric albedo).

We find that a large majority of JF comets with perihelion distances $q > 2.5$ AU are brighter than absolute nuclear magnitude $H_N = 16$, suggesting that only a very small fraction (a few percent) of the population of the JF comets with large q has so far been detected. A similar trend is noted for the corresponding absolute total magnitudes H_T taken from Kresák & Kresáková's (1994) catalog. By analyzing the H_N and H_T data, and trends in the discovery rate of JF comets as a function of their perihelion distances, the overall population of JF comets within Jupiter's region ($q < 5.2$ AU) up to an absolute nuclear magnitude $H_N = 18.5$ is estimated to be from several thousand to about 10^4 members. The q-distribution of JF comets shows a steep increase with q, which is consistent with JF comets coming from a flat intermediate source in the Jupiter-Saturn region.

Key words: astronomical data bases: miscellaneous – comets: general

1. Introduction

Comets of the Jupiter Family (JF) constitute a special sub-set within the observed comet population: their dynamical evolution is controlled by Jupiter and all move in direct orbits, generally very close to the ecliptic plane, in contrast to the random distribution of the orbital planes of long-period comets. The Tisserand constant T is a convenient parameter to distinguish JF comets from the Halley-type comets or long-period comets (Valsecchi 1992). JF comets have $T > 2$ while Halley-type and LP comets have $T < 2$ (Fernández 1994). All the discovered JF comets have orbital periods $P < 20$ yr, while only four comets with $P < 20$ yr do no fulfill the requirement of $T > 2$. These are: 96P/Machholz 1, 126P/IRAS, 8P/Tuttle and P/1994 X1 (McNaught-Russell) which have not been included into our sample.

The physical study of JF comets and estimates about their steady-state number in the inner planetary region (interior to Jupiter's orbit) may help to better understand their origin and their physical and dynamical evolution, as well as their contribution to the impact cratering record of the surfaces of the terrestrial planets and the Moon (e.g. Weissman 1990, Bailey 1991). Basic data as, for instance, their typical size and size distribution are of fundamental importance to this purpose, but unfortunately they are very difficult to evaluate, mainly due to the fact that comet nuclei are generally surrounded by comae of gas and dust. There has nevertheless been a growing effort to measure comet magnitudes at ever larger heliocentric distances despite the difficulties involved. At distances greater than ~ 3 AU JF comets generally show a low level of activity, so most of the measured magnitudes may be close to nuclear (Kresák 1973). In most cases "nuclear" should be understood

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as the magnitude of the comet nucleus plus a certain coma contamination. It is expected that, as comets are observed farther away from the Sun, the nuclear magnitudes will approach the actual magnitudes of comet nuclei.

Among the few systematic observers of cometary nuclear magnitudes, we can mention Elizabeth Roemer who carried out a long-term observing program that provided nuclear magnitudes for a large sample of JF comets measured on photographic plates, taken with the 40-inch, f/6.8 Ritchey-Chrétien reflector at Flagstaff. During the last decade the data set of nuclear magnitudes has been greatly enlarged and improved by using CCD detectors attached to medium- and large-size telescopes. Particularly relevant is the long-term observing program of distant JF comets carried out by James Scotti and colleagues with the Spacewatch telescope. There has also been near-aphelion, CCD photometry of a few JF comets, such as 2P/Encke (Jewitt & Meech 1987) and 31P/Schwassmann-Wachmann 2 (Luu & Jewitt 1992a). A few other, presumably low-active JF comets were observed photometrically and spectroscopically in the infrared (sometimes combined with the visible), such as 49P/Arend-Rigaux (Brooke & Knacke 1986) and 28P/Neujmin 1 (Campins et al. 1987). All this pioneering work was aimed at establishing a first physical picture of a comet nucleus, including its size, albedo, fraction of active surface area and rotation period. A few observers have embarked upon programs of CCD photometric measurements of distant comets (David Jewitt, Karen Meech, Carl Hergenrother, Steve Larson, among others). There have also been some observations with the Hubble Space Telescope (e.g. Lamy & Toth 1995) and, finally, we have our own program in operation since 1990 which has by now provided measurements of nuclear magnitudes of 12 JF comets (Licandro et al. 1999).

For a few periodic comets we have now fairly good estimates of their size and albedo. This is the case of 1P/Halley observed *in situ* by the Giotto and Vega spacecraft, and a few others observed from the ground in the visible and/or near infrared (see, e.g., Jewitt 1991 for a review). Almost all periodic comets so far studied have very low visual geometric albedos ($p_v \approx 0.02$–0.05) with an average value $p_v \sim 0.04$ (Hartmann et al. 1987).

We will first analyze the concept of "nuclear" and "total" magnitude and their relationship, and the observed material. The data set of nuclear magnitudes of JF comets determined by different authors, including ours, and the best estimates of the absolute nuclear magnitudes with their uncertainties are discussed by Tancredi et al. (1999) (hereafter referred to as Paper I). The data set of absolute total magnitudes has been taken from Kresák & Kresáková's (1994) catalog. We will next discuss size, size distribution, the distribution of perihelion distances and the size of the overall JF population in the region interior to Jupiter's orbit.

2. The data set of nuclear magnitudes

About half of the magnitudes discussed in Paper I were collected by Kamél (1991) from different sources. Kamél's set comprises all nuclear magnitudes plus the total magnitudes observed at heliocentric distances > 3 AU. His catalog is complete through the end of 1989. A new catalog of nuclear magnitudes was produced in 1991 by Fernández et al. (1992). Paper I updates these two previous catalogs by adding measurements of nuclear magnitudes for the period: 1990 – mid-1998. The selection criteria, details on color and phase corrections, and results of the new catalog are presented in Paper I.

From the observed apparent nuclear magnitudes m_N we have derived the absolute nuclear magnitude H_N through the usual expression:

$$H_N = m_N - 5 \log r\Delta - 0.04\alpha,$$

where r is the heliocentric distance, Δ the geocentric distance, and α the phase angle measured in degrees. Therefore, the absolute nuclear magnitude is the magnitude the comet would present if it were located at $r = \Delta = 1$ AU and phase angle $\alpha = 0$, namely $H_N \equiv m_N(1, 1, 0)$.

Recent determinations of absolute nuclear magnitudes, H_N, tend to be fainter as JF comets are being observed at ever larger heliocentric distances where they show very low activity. However, there are some JF comets whose derived values of H_N show a large spread at large heliocentric distances including very bright values. Comets 2P/Encke, 28P/Neujmin 1, 39P/Oterma, and 10P/Tempel 2 show this kind of anomalous behavior. It is possible that some JF comets maintain some residual activity at heliocentric distances as far as 6–7 AU, so their measured nuclear magnitudes still have some coma contamination. Part of the brightness variations may also be intrinsic, due to the rotation of a very elongated nucleus.

In Paper I we plotted the absolute nuclear magnitudes of a given JF comet, measured by different authors, as a function of the heliocentric distances at which they were determined. From such a plot we obtained our "best" estimate of H_N following certain criteria discussed there. The estimated nuclear magnitudes of 105 JF comets of Paper I are shown in Table 1 together with their quality classes (QC). Nuclear magnitudes of $QC = 1$ are the most reliable ones, and their uncertainty can be estimated within about ± 0.3 mag. Nuclear magnitudes of $QC = 2$ may have an uncertainty between about ± 0.3 and ± 0.6 mag. $QC = 3$ corresponds to uncertainties between about ± 0.6 and ± 1 magnitudes. Finally, $QC = 4$ indicates very poorly estimated magnitudes, because they rely upon old data, few observations or observations showing a large scatter. They have an uncertainty generally well above ± 1 mag or can be considered only as a lower limit (i.e., the comet should be fainter than the quoted value).

The absolute nuclear magnitudes of JF comets, H_N, are plotted in Fig. 1 vs. the perihelion distances q. The trend of absolute nuclear magnitudes to be on average brighter for comets with larger q is confirmed (Fernández et al. 1992), suggesting a low degree of completeness of the sample of distant JF comets. It is also noticeable that no comets fainter than $H_N \sim 19.5$ are found in the sample, even for small-q comets where the sample presumably is more complete. Whether this lack of observed faint JF comets is a real effect or an artifact of selection effects

Table 1. Absolute nuclear magnitudes of JF comets

Comet	H_N	QC	Comet	H_N	QC
50P/Arend	15.2	4	97P/Metcalf-Brewington	17.0	4
49P/Arend-Rigaux	15.1	1	P/1997G1 (Montani)	15.6	4
47P/Ashbrook-Jackson	15.3	1	124P/Mrkos	16.6	2
19P/Borrelly	15.2	1	120P/Mueller 1	18.1	3
P/1992Q1 (Brewington)	16.8	4	131P/Mueller 2	18.2	4
16P/Brooks 2	16.5	3	P/1990S1 (Mueller 3)	16.2	4
87P/Bus	17.1	3	P/1993W1 (Mueller 5)	16.0	4
101P/Chernykh	15.9	3	28P/Neujmin 1	12.8	1
67P/Churyumov-Gerasimenko	15.6	1	42P/Neujmin 3	18.7	4
71P/Clark	17.1	2	39P/Oterma	12.8	4
32P/Comas-Sola	15.6	3	119P/Parker-Hartley	15.6	4
33P/Daniel	17.9	4	7P/Pons-Winnecke	16.8	3
6P/D'Arrest	16.7	2	30P/Reinmuth 1	17.1	3
57P/Du-Toit-Neujmin-Delporte	16.6	4	44P/Reinmuth 2	16.8	4
2P/Encke	17.0	3	89P/Russell 2	17.4	3
4P/Faye	15.9	2	91P/Russell 3	17.1	4
15P/Finlay	17.9	4	94P/Russell 4	16.2	3
37P/Forbes	17.6	3	24P/Schaumasse	18.0	4
90P/Gehrels 1	15.4	4	29P/Schwassmann-Wachmann 1	12.0	2
78P/Gehrels 2	16.0	3	31P/Schwassmann-Wachmann 2	15.1	1
82P/Gehrels 3	16.1	2	73P/Schwassmann-Wachmann 3	17.7	4
P/1997C1 (Gehrels 4)	15.8	3	61P/Shajn-Schaldach	17.5	3
21P/Giacobini-Zinner	17.7	2	P/1994J3 (Shoemaker 4)	15.0	4
84P/Giclas	16.9	4	128P/Shoemaker-Holt 1	16.1	4
26P/Grigg-Skjellerup	17.1	1	P/1990UL3 (Shoemaker-Levy 2)	15.3	2
65P/Gunn	14.2	2	118P/Shoemaker-Levy 4	16.5	3
51P/Harrington	16.9	4	P/1991V2 (Shoemaker-Levy 7)	18.0	4
52P/Harrington-Abell	17.4	4	P/1992G2 (Shoemaker-Levy 8)	16.8	3
100P/Hartley 1	17.0	4	105P/Singer-Brewster	17.7	4
103P/Hartley 2	14.7	4	56P/Slaughter-Burnham	16.7	3
110P/Hartley 3	16.2	2	74P/Smirnova-Chernykh	13.7	4
P/1993K2 (Helin-Lawrence)	14.3	4	125P/Spacewatch	18.0	2
117P/Helin-Roman-Alu 1	14.9	2	113P/Spitaler	17.4	4
132P/Helin-Roman-Alu 2	17.8	4	64P/Swift-Gehrels	16.4	4
111P/Helin-Roman-Crockett	16.7	3	98P/Takamizawa	15.7	4
17P/Holmes	16.1	3	9P/Tempel 1	15.8	1
45P/Honda-Mrkos-Pajdušáková	19.3	4	10P/Tempel 2	15.3	2
88P/Howell	17.4	3	62P/Tsuchinshan 1	18.1	4
58P/Jackson-Neujmin	18.7	4	60P/Tsuchinshan 2	18.2	3
P/1995A1 (Jedicke 1)	15.2	3	41P/Tuttle-Giacobini-Kresák	18.5	3
P/1996A1 (Jedicke 2)	14.1	2	112P/Urata-Niijima	18.4	4
48P/Johnson	15.9	2	40P/Väisälä 1	16.7	3
68P/Klemola	15.9	3	53P/Van-Biesbroeck	14.7	3
75P/Kohoutek	16.3	3	123P/West-Hartley	15.9	4
70P/Kojima	17.3	3	76P/West-Kohoutek-Ikemura	17.1	3
22P/Kopff	16.3	2	36P/Whipple	15.8	2
99P/Kowal 1	14.2	3	81P/Wild 2	15.9	3
P/1983J3 (Kowal-Vavrova)	16.9	3	86P/Wild 3	17.8	3
P/1994A1 (Kushida)	17.3	3	116P/Wild 4	14.9	3
P/1993X1 (Kushida-Muramatsu)	15.8	3	46P/Wirtanen	18.4	1
P/1997V1 (Larsen)	14.8	4	14P/Wolf	17.0	4
77P/Longmore	15.7	4	43P/Wolf-Harrington	16.3	2
130P/McNaught-Hughes	16.5	3			

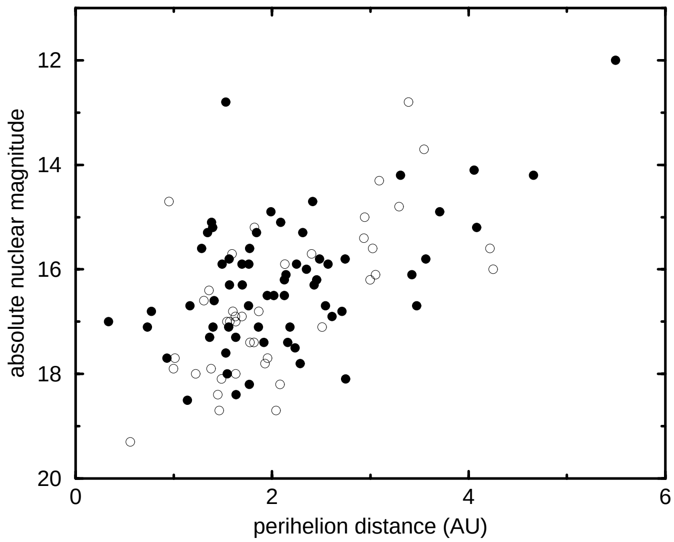

Fig. 1. Absolute nuclear magnitudes versus perihelion distances for the sample of JF comets shown in Table 1. Filled circles are for comets of quality classes 1 to 3. Open circles are for comets of quality class 4.

is not possible to ascertain yet. It might be possible that small comets that come into the inner planetary region undergo a fast disintegration process into meteoroidal dust (Fernández et al. 1999).

3. Nuclear and total magnitudes

To gauge the reliability of the determined nuclear magnitudes m_N it is worth comparing them with the respective total magnitudes m_T. Differences between m_N and m_T can reach values in the range of 5–10 magnitudes for a given JF comet. Let us derive theoretically a relation between m_N and m_T assuming that the coma brightness is produced by light scattered by dust particles. Let Q_d be the dust production rate (in number of particles s^{-1}). The dust particles leave the comet's gravitational influence zone with a terminal velocity v_d. For a spherically symmetrical dust coma with uniform radial outflow, the number density of dust particles, $n_d(\nu)$, at a radial distance ν can be obtained as

$$n_d(\nu) = \frac{Q_d}{4\pi\nu^2 v_d} \tag{1}$$

The column density of dust particles at a projected distance x from the nucleus will thus be expressed as

$$N_d(x) = \int_{-\infty}^{+\infty} n_d(\nu) ds = \int_{-\infty}^{+\infty} \frac{Q_d}{4\pi\nu^2 v_d} ds = \frac{Q_d}{4 v_d x} \tag{2}$$

where the distance ds is taken along the line of sight, and $\nu^2 = x^2 + s^2$.

Let a be the typical radius of the dust particles that contribute most to the brightness of the dust coma; it should be of the order of optical wavelengths, i.e. $a \sim 0.5\,\mu$m (Luu & Jewitt 1992b). The brightness B_A (per unit of area) of the dust coma will be proportional to the column density of dust particles; therefore, according to Eq. (2), it decreases with the distance x from the comet nucleus and is given by

$$B_A = \frac{Q_d b_a}{4 v_d x} \tag{3}$$

where b_a is the brightness of a dust particle that depends on its scattering efficiency. If the comet is at a geocentric distance Δ, the relation between sky area and subtended solid angle is $dA = \Delta^2 d\Omega$, so the brightness per unit solid angle is

$$B_\Omega = B_A \frac{dA}{d\Omega} = \frac{Q_d b_a \Delta^2}{4 v_d x} = \frac{Q_d b_a \Delta}{4 v_d \xi} \tag{4}$$

where $\xi = x/\Delta$ is the angular distance from the center to the considered area. Note that Eq. (4) predicts: (a) a dropoff of the observed brightness as ξ^{-1}, which is in accordance with observations of dust comae showing a sharp central condensation; (b) a brightness for otherwise equal circumstances that increases proportionally to Δ, leading to a dimming of observed comae as comets come close to Earth. However, this is more than compensated by the increasing brightness of each individual grain given by

$$b_a = c_\odot \frac{\pi a^2 r_\oplus^2}{r^2} \frac{p_a \phi_a(\alpha)}{\Delta^2} \tag{5}$$

where $c_\odot = 1360$ W m^{-2} is the solar constant, $r_\oplus = 1.5 \times 10^{11}$ m is the radius of the Earth's orbit, p_a is the geometric albedo and $\phi_a(\alpha)$ is the phase function of the grain at phase angle α. Infrared observations in the range 1–20 μm of comet 27P/Crommelin show that dust grains are very dark, with geometric albedos as low as 0.015 in the J band (Hanner et al. 1985a). These observations refer to large, fluffy particles a few microns in size or larger, while we are concerned with smaller grains, probably the disintegration products of large, fluffy grains. Since the fluffy structure favors absorption, more compact smaller grains may have somewhat greater albedos. Indeed, variations in the albedo across the coma are found in several comets (Sekanina et al. 1998). Bearing this in mind we will adopt a value $p_a\phi_a(\alpha) = 0.03$, assuming that the comet is observed at small phase angles.

The radius of the coma will be set either by the distance x_s at which the coma brightness fades into the sky background, or by the distance x_{rp} at which the dust particles are dragged into the dust tail by the solar radiation pressure, or by the field of view of the instrument employed. Let us consider first the case where an observer sees the coma gradually fading away into the sky background as ξ increases. The limit will be reached for a certain ξ_s beyond which the signal to noise ratio will no longer be significant. From Eq. (4) we get

$$\xi_s = \frac{Q_d b_a \Delta}{4 v_d B_{\Omega s}} \tag{6}$$

where $B_{\Omega s}$ is the minimum brightness (per unit sky area) that can be detected against the sky background. This limit will depend on the instrument employed. For instance, an amateur observing with binoculars with a typical seeing of $3''$ could reach sources as faint as $\sim 12th$ magnitude, which leads to a threshold of $B_{\Omega s} \sim 14.1$ mag arcsec^{-2}, whereas a professional astronomer observing with a large telescope and CCD camera usually reaches ~ 21.5 mag with a typical seeing $\sim 1.5''$, implying a threshold $B_{\Omega s} \sim 22.1$ mag arcsec^{-2}.

Due to the solar radiation pressure, small dust grains will reverse the direction of motion thus creating an enhanced grain density in the coma. Let $\gamma_1 (\geq 1)$ be a factor that takes this effect into account. Appropriate numerical values will be discussed below.

The integrated coma brightness will thus be obtained as

$$B_{coma} = \int_0^{\xi_s} \frac{Q_d \gamma_1 b_a \Delta}{4 v_d \xi} 2\pi\xi d\xi = \frac{\pi Q_d \gamma_1 b_a \xi_s \Delta}{2 v_d} \quad (7)$$

By subtituting Eqs. (4) and (6) into Eq. (7) we finally get

$$B_{coma} = \frac{\pi}{2 B_{\Omega s}} \left(\frac{Q_d \gamma_1 c_\odot r_\oplus^2 \pi a^2 p_a \phi_a(\alpha)}{2 v_d} \right)^2 r^{-4} \Delta^{-2} \quad (8)$$

The angular diameter of the coma will simply be

$$\begin{aligned} \Phi_{coma} &= 2\xi_s = \frac{Q_d \gamma_1 b_a \Delta}{2 v_d B_{\Omega s}} \\ &= \frac{Q_d \gamma_1 c_\odot r_\oplus^2 \pi a^2 p_a \phi_a(\alpha)}{2 v_d B_{\Omega s}} r^{-2} \Delta^{-1} \end{aligned} \quad (9)$$

where we have substituted b_a by Eq. (5).

Note that Eq. (8) predicts no "Delta effect", i.e. deviation from a Δ^{-2} law, for comets in general. Observed Delta effects are controversial and anyway limited to very small geocentric distances (Kamél 1991). Eq. (8) also predicts a r^{-4} dependence of brightness supplemented by a decrease of Q_d^2 with r. The lightcurves would hence in general be steeper than the standard r^{-4} law, which seems to be supported by the observational data of short-period comets (Whipple 1978). Eq. (9) predicts a general trend of coma diameters to decrease rapidly with increasing r (and Δ). The observed trend of coma diameters to shrink as comets approach the Sun inside Earth's orbit may be due to: (1) importance of gaseous species whose lifetimes decrease as r decreases; (2) a deterioration of the sky background such that $B_{\Omega s}$ increases (so ξ_s decreases).

If the solar radiation pressure sets the limit of the coma radius, this can be roughly computed as the distance at which a dust particle released in the solar direction with a terminal velocity v_d, comes to a halt by the radiation pressure (and starts to reverse its motion towards the dust tail). This distance, x_{rp}, is easily computed by (see, e.g., Grün & Jessberger 1990)

$$x_{rp} = \frac{v_d^2 r^2}{2\beta G M_\odot} \quad (10)$$

where G is the gravitational constant, $M_\odot$ is the Sun's mass, and β is the ratio of the force associated with the Sun's radiation pressure to the gravitational force of the Sun. It is given by

$$\beta = 5.78 \times 10^{-4} \frac{\eta_{pr}}{\rho_d a} \text{ kg m}^{-2} \quad (11)$$

where η_{pr} is the radiation pressure efficiency and ρ_d is the mass density of the dust particles. $\eta_{pr} \sim 1$ for grain sizes comparable to the wavelengths of visible light, and we can adopt an average mass density $\rho_d = 2\,\text{g}\,\text{cm}^{-3}$ that is in agreement with the mass and bulk density determination of dust particles released by 1P/Halley (Maas et al. 1990).

In this case the integrated coma brightness will be given by

$$B_{coma} = \frac{\pi Q_d \gamma_2 b_a x_{rp}}{2 v_d} \quad (12)$$

where γ_2 is again a factor that takes into account an enhanced grain density in the coma by solar radiation pressure.

Substituting b_a by Eq. (5) and x_{rp} by Eq. (10), we get

$$B_{coma} = \frac{\pi^2 Q_d \gamma_2 c_\odot r_\oplus^2 a^2 p_a \phi_a(\alpha) v_d}{4\beta G M_\odot} \Delta^{-2} \quad (13)$$

As seen, in this case the coma brightness does not depend explicitly on r; it only depends implicitly on r through Q_d.

The coma diameter is in this case

$$\Phi_{coma} = 2\frac{x_{rp}}{\Delta} = \frac{v_d^2}{\beta G M_\odot} r^2 \Delta^{-1} \quad (14)$$

Let us now assume that the limit to the coma extent is set by the field of view of the instrument employed. Let ξ_i be the limit in this case. Typically, total magnitudes of comets close to the Sun are obtained by amateurs with binoculars or wide-field telescopes. We can assume in this case a field of view of $\xi_i \sim 2^o$. Total magnitudes of distant comets are mainly obtained with CCD cameras attached to large focal ratio telescopes. We assume in the latter case a much smaller $\xi_i \sim 3$ arcmin. The coma brightness becomes

$$\begin{aligned} B_{coma} &= \frac{\pi Q_d \gamma_3 b_a \xi_i \Delta}{2 v_d} \\ &= \frac{\pi^2 Q_d \gamma_3 c_\odot r_\oplus^2 a^2 p_a \phi_a(\alpha) \xi_i}{2 v_d} r^{-2} \Delta^{-1} \end{aligned} \quad (15)$$

where γ_3 is the enhancement factor by solar radiation pressure.

In this case the coma diameter is simply given by

$$\Phi_{coma} = 2\xi_i \quad (16)$$

Let us analyze suitable values for γ_1, γ_2 and γ_3. In the case that the coma diameter is set by the solar radiation pressure, we can argue that the dust particles leaving the nucleus in the hemisphere facing the Sun will reverse their motion to the coma region, thus increasing the grain population in the coma by a factor $\gamma_2 \sim 1.5$. In the case that the coma diameter is set either by the fading into the sky background or by the telescope's field of view, only a fraction of the dust particles reversing motion will pass again through the visible coma, so the enhancement factors have to be scaled down to values $\gamma_1 \sim 1 + 0.5 \times (\Phi_{coma,1}/\Phi_{coma,2})^2$ and $\gamma_3 \sim 1 + 0.5 \times (\Phi_{coma,3}/\Phi_{coma,2})^2$, respectively, where $\Phi_{coma,1}$, $\Phi_{coma,2}$ and $\Phi_{coma,3}$ are the coma diameters given by Eqs. (9), (14) and (16), respectively.

The total brightness of the comet, B_T, will be expressed as

$$B_T = B_{coma} + B_N \quad (17)$$

where B_N is the brightness of the comet nucleus given by

$$B_N = c_\odot \frac{\pi R_N^2 r_\oplus^2}{r^2} \frac{p_N \phi_N(\alpha)}{\Delta^2} \quad (18)$$

Here R_N is the radius, p_N the geometric albedo, and $\phi_N(\alpha)$ the phase function at phase angle α of the comet nucleus.

If we consider the corresponding apparent total and nuclear magnitudes, we obtain

$$m_N - m_T = 2.5 \times \log \left(1 + \frac{B_{coma}}{B_N}\right) \quad (19)$$

If we introduce either Eq. (8), or Eq. (13), or Eq. (15) for B_{coma} and Eq. (18) for B_N into Eq. (19) we obtain the difference between the apparent total and nuclear magnitude as a function of r and Δ.

The terminal velocity of dust particles v_d is approximately $\propto (ZR_N/\rho_d a)^{1/2}$ (Hanner et al. 1985b), where $Z = Z(r)$ is the gas production rate per unit of area (Fernández et al. 1999). It is interesting to stress the proportionality of v_d to $R_N^{1/2}$; in particular $v_d \sim 0.45\,\mathrm{km\,s^{-1}}$ for a typical grain radius $a = 0.5\,\mu$m and for $R_N = 1$ km (Hanner 1985). Therefore we will adopt in the following $v_d = 450[Z/Z_o\ R_N]^{1/2}$ m s^{-1}, where Z_o is the gas production rate per unit area at $r = 1$ AU and R_N is expressed in km.

We can relate the dust production rate to the gas production rate, assuming the latter to arise mainly by the sublimation of water ice. We have

$$Q'_d = \frac{\psi}{1+\psi} Q_{H2O} \quad (20)$$

where $Q'_d = \frac{4}{3}\pi a^3 \rho_d Q_d$ is the dust production rate expressed in kg s^{-1}, and ψ is the ratio of the dust to gas production rates, $\psi \sim 0.5$ (Hanner 1985). The total water mass production rate is given by

$$Q_{H2O} = 4\pi R_N^2 f Z m_{H_2O} \quad (21)$$

where f is the fraction of active surface area of the comet nucleus, and m_{H_2O} is the mass of a water molecule.

We note that ψ as well as f may vary with heliocentric distance for the same comet, since the comet nucleus may suffer a cyclical process of partial formation and expulsion of the dust mantle as the comet recedes from and approaches the Sun. We also note that for distances $r \gtrsim 3.5$ AU, sublimation of CO_2 or CO may control the gas production rate and drive the activity of distant comets (Senay & Jewitt 1994; Jewitt et al. 1996), thus leading to a different ratio of the dust to gas production rates. Therefore, in computing $m_N - m_T$ for a given comet, we should bear in mind the possible dependence of ψ and f on the heliocentric distance though, for the time being, given the complexity of the problem we have assumed these parameters to be constant along the comet's orbit.

Fig. 2 shows the difference between the apparent nuclear and total magnitudes computed from Eq. (19), assuming that the comet is at opposition, i.e. $\Delta = r - r_\oplus$, for comet nuclei of $R_N = 1$ km and $R_N = 5$ km, as a function of the heliocentric distance and for different fractions f of active surface areas. For the computations we have assumed the case of telescopic observations with CCD camera, thus setting the limits for the sky background and field of view at 22.1 mag arcsec^{-2} and 3 arcmin, respectively. As seen in the figure, the curves show different slopes for different heliocentric distances: they are flat or slightly positive for small r where the field of view regime dominates in defining the size of the visible coma. The slope is strongly negative, i.e. there is a strong dropoff in the magnitude difference with r for larger r (or very small f), where the sky background regime dominates. The radiation pressure regime dominates for large nuclei with large f, and for large r; more specifically, the portions with negative slope of the two curves with $f = 1$; and the distant portion of the curve ($r \gtrsim 3.4$ AU) for $R_N = 5$ km and $f = 0.1$.

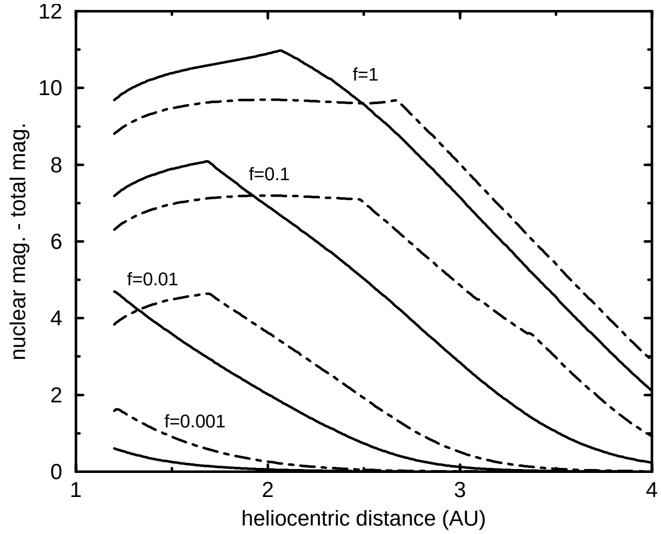

Fig. 2. Computed difference between the apparent nuclear and total magnitudes as a function of the heliocentric distance, for a 1-km size (solid curves) and a 5-km size (dot-dashed curves) comet nucleus, and for different fractions f of active surface areas.

We can see that differences can amount to about 10 magnitudes for very active comets at distances $r \lesssim 2$ AU, but the difference becomes negligible (i.e. the total magnitude reduces to the nuclear magnitude), for distances $r \gtrsim$ 3–4 AU and $f \lesssim 0.1$. For $r \gtrsim 3.5$ AU the theoretical results would indicate that m_T comes close to m_N for a typical comet nucleus of one to a few km size and a small fraction of active surface area ($f \lesssim 0.1$). This is in agreement with Kresák's (1973) consideration that the bare nucleus is virtually observed for $r > 3$ AU. However, our results show that comets with larger f may still be quite active which may explain the persistent activity of some JF comets at large heliocentric distances. For instance, we have already discussed the fact that some JF comets show strong fluctuations in their measured nuclear magnitudes at even larger heliocentric distances, suggesting that an important coma contribution is still present in these comets. We stress that large uncertainties should be attached to our computed results of Fig. 2, mainly for $r \gtrsim 2$ AU, as discussed above, so they should only be taken as a first approach to the problem.

The uncertainty of the measured nuclear magnitudes could be larger for small, faint JF comets generally observed close to the Sun, where the coma contribution largely overcomes the brightness contribution from the bare nucleus. For these comets the estimate of their nuclear magnitudes necessarily rests on a procedure of the type adopted by Scotti to substract the coma contribution, which may be rather uncertain, particularly when the coma is several magnitudes brighter than the nucleus.

Since generally $B_{coma} \gg B_N$, we have $B_T \sim B_{coma}$. From Eqs. (20) and (21) we have $Q_d \propto R_N^2$. Furthermore, $v_d \propto R_N^{1/2}$. If the limit of the coma is set by its fading into the sky background – applicable to more distant and smaller comet nuclei – we get from Eq. (8)

$$B_T \propto \frac{Q_d^2}{v_d^2} \propto R_N^3 \quad (\mathrm{Low-Active\ Comet}) \tag{22}$$

For comets with a large fraction of free sublimating area, far from the Sun, the limit of the coma is set by the solar radiation pressure, so from Eq. (13) we get

$$B_T \propto Q_d \times v_d \propto R_N^{5/2} \quad (\mathrm{Active\ Comet}) \tag{23}$$

If the limit is set by the field of view of the instrument employed, which will generally apply to active comets close to the Sun, we get from Eq. (15)

$$B_T \propto Q_d / v_d \propto R_N^{3/2} \quad (\mathrm{Active\ Comet}) \tag{24}$$

It is interesting to note the different dependence of the total brightness on R_N for the different regimes. If we consider magnitudes instead of brightness and bear in mind that the nuclear brightness $B_N \propto R_N^2$, we get the corresponding relations

$$m_T = C_1 + 1.50 m_N \quad (\mathrm{Low-Active\ Comet}) \tag{25}$$

$$m_T = C_2 + 1.25 m_N \quad (\mathrm{Active\ Comet}) \tag{26}$$

or

$$m_T = C_3 + 0.75 m_N \quad (\mathrm{Active\ Comet}) \tag{27}$$

where C_1, C_2 and C_3 are constants.

The derived slopes will be valid for absolute total and nuclear magnitudes: H_T and H_N. The absolute total magnitude H_T is defined as the total magnitude the comet would have if it were located at 1 AU to the Earth and to the Sun, assuming a brightness variation law r^n, where usually $n = 4$.

Absolute total and nuclear magnitudes of the observed JF comets are plotted in Fig. 3 for four different ranges of q. The absolute total magnitudes have been taken from Kresák & Kresáková's (1994) catalog. The four solid curves are for our theoretical relation (H_N, H_T) for fractions of free sublimating surface area f = 0.01, 0.1, 0.3 and 1.0. We can obtain the absolute nuclear magnitude H_N as a function of the nucleus radius R_N and for a given geometric albedo: $p_v = 0.04$. The photometric cross-section S of a nucleus of radius R_N is given by

$$\log(p_v S) = 16.85 + 0.4 \times [m_\odot - H_N] \tag{28}$$

where $S = \pi R_N^2$ is expressed in km^2, and $m_\odot = -26.77$ is the apparent (visual) magnitude of the Sun. From H_N we can obtain m_N from the equation $m_N = H_N + 5 \log r\Delta + 0.04\alpha$, where average values of r, Δ and α were computed for each range of q by assuming that comets are discovered when they are at their perihelia. We can also get the average configuration Sun–Earth–comet at the time of discovery by taking the median of $\cos(\lambda - \lambda_\odot)$ and $\cos\beta$, where λ, $\lambda_\odot$ are the longitudes of the comet and the Sun, respectively, and β is the latitude of the comet, taken from Kosai & Nakamura (1991). From the average ecliptic coordinates of the comet at the time of discovery (where it is assumed to be at perihelion), we obtain the average r, Δ and α. Once we get m_N, we can obtain m_T from Eq. (19). Finally, we obtain H_T through the usual expression $H_T = m_T - 10 \log r - 5 \log \Delta$.

Most points fall in the region of $0.01 < f < 0.3$ which would suggest that most JF comets have fractions of active surface areas not larger than a few tenths. A few points fall in the forbidden region $f > 1$ which is probably due to the several uncertainties involved in our theoretical model and the derived total and nuclear magnitudes. It is particularly striking to find that all four comets in the range $2.75 < q < 3.25$ AU fall in the forbidden region $f > 1$. This might be an indication that water ice does not control any longer the gas production rate of these comets, so most of their gaseous activity should arise from another more volatile substance such as CO or CO_2.

We can compare our theoretical results with the empirical relation derived by Jorda et al. (1992)

$$m_T = 128.1\text{–}4.17 \times \log Q_{H2O} \tag{29}$$

where Q_{H2O} is expressed in mol s^{-1}. From Eq. (21) we have $Q_{H2O} = 4\pi R_N^2 f Z$, and $H_N = A - 2.5 \log R_N^2$, so introducing this equation into Eq. (29), we can get after some manipulation an expression of the type: $H_T = B + 4.17/2.5 H_N \simeq B + 1.67 H_N$, where A and B are constants. Therefore, the empirical law derived by Jorda et al. is close to our theoretical estimate for the sky background regime [Eq. (25)], though significantly steeper than that for the field of view regime. Four curves derived from the Jorda et al. relation for f = 0.01, 0.1, 0.3 and 1.0 are also shown in Fig. 3. As seen the fit is rather poor: in this case a significant fraction of comets are left in the forbidden region $f > 1$, mainly for large q.

4. The distributions of absolute nuclear magnitudes and masses

Up to the present, estimates of the mass distribution of comets have generally relied upon the distribution of absolute total magnitudes H_T (e.g. Hughes 1988, Bailey 1990). Such estimates are quite uncertain bearing in mind that H_T is a poorly defined quantity that depends on the assumed slope n of the heliocentric comet's lightcurve ($n = 4$ is usually assumed which leads to the well-known total magnitude H_{10}). Furthermore, the mass-magnitude relationship is very uncertain (Bailey 1990). More discussion on the distribution of H_T will be left to the next section. We shall concentrate here on the derivation of the mass

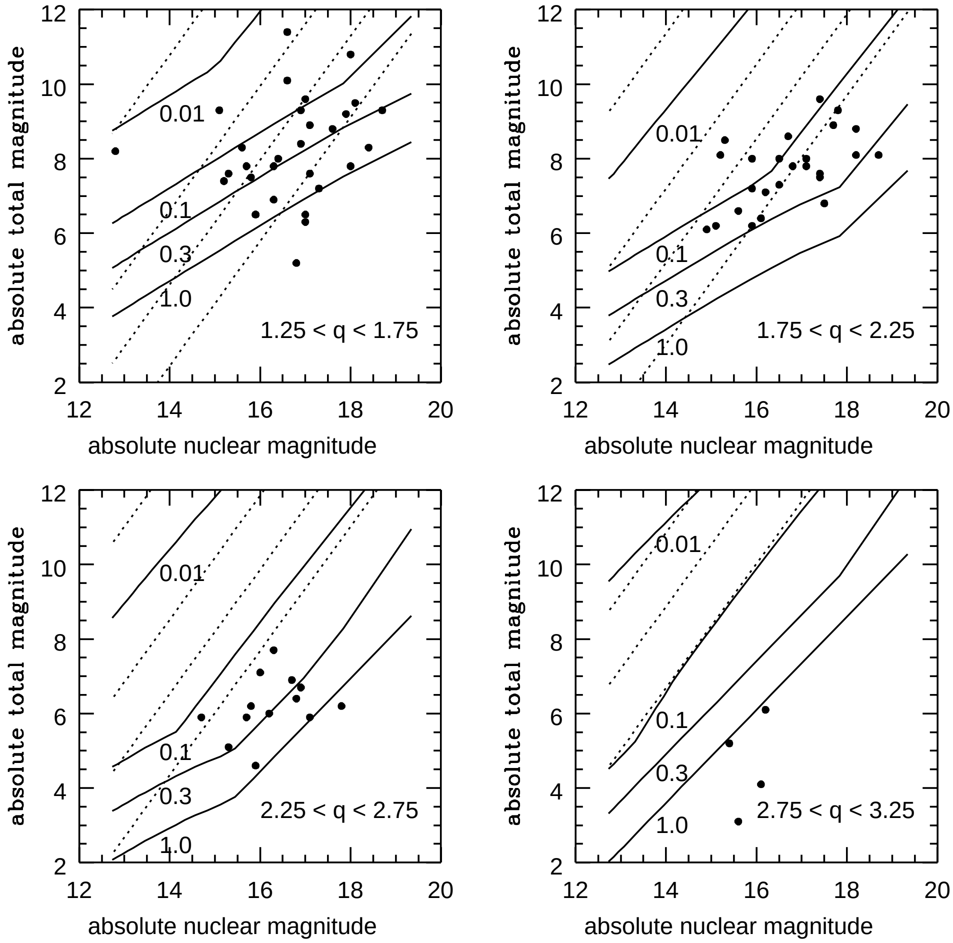

Fig. 3. Absolute total magnitudes versus absolute nuclear magnitudes of the observed JF comets within the four ranges of perihelion distances indicated on the plots. The solid curves are obtained from our theoretical relation derived as described in the text, for four values of the free-sublimating area indicated beside each curve. The dotted curves are obtained from the Jorda et al. (1992) empirical relation for the same values of f.

distribution law for the JFC population from the distribution of nuclear magnitudes, which may be more reliable and straightforward. In this regard, Shoemaker & Wolfe (1982) used the sample of Roemer's data of nuclear magnitudes to derive a linear relationship $\log N_N(H_N) = C + 0.4H_N$, up to the absolute blue magnitude 16, where $N_N(H_N)$ is the cumulative number of comets brighter than H_N and C is a constant. Let us re-discuss this issue from the data presented in Table 1.

We have analyzed the cumulative distribution of absolute nuclear magnitudes for JF comets with $q < 2$ AU, for which we can assume that the sample of bright comets is more complete. The luminosity function follows a linear relation up to $H_N \sim 16.0$. For $H_N \gtrsim 16.0$ the distribution tends to flatten due probably to incompleteness of comet discoveries. For $H_N \lesssim 16.0$ we obtain a slope ~ 0.55, whereas if we remove the comets of QC=4 the slope decreases to 0.54 (Fig. 4). If we limit the sample to $q < 1.5$ AU we get the corresponding slopes ~ 0.48 (for all quality classes) and 0.55 (without QC=4). The results are still rather uncertain which is due to the small number of comets involved in the determination of the slope. From the analysis of these cases we can estimate an index $\sim 0.53 \pm 0.05$. Thus, our H_N-distribution turns out to be somewhat steeper than that derived by Shoemaker & Wolfe.

Bearing in mind that $H_N = k - 2.5 \log B_N$ and that $B_N \propto R_N^2 \propto M^{2/3}$, from the cumulative distribution of absolute nuclear magnitudes we can derive the cumulative mass distribution

$$n_c(M) \propto M^s \tag{30}$$

where $s \simeq -0.88 \pm 0.08$.

A power-law distribution of index -0.88 turns out to be somewhat steeper than the theoretical distribution of index $-5/6$ expected for self-similar collision cascades as predicted by Dohnanyi (1969) for main-belt asteroids. Cellino et al. (1991) used the IRAS data base on albedos and diameters to derive size distributions for a set of about 4000 main-belt asteroids. They found a bimodal distribution with a slope $s \sim -1$ for diameters $D \gtrsim 150$ km, while smaller asteroids (down to the completeness limit diameter of 44 km) show a much flatter distribution with $s \sim -0.35$. From a sample of more than 60,000 asteroids found with the Spacewatch survey between 1992 and 1995, Jedicke & Metcalfe (1998) also found a rather flat distribution with $s \sim -0.5$ for small asteroids of a few km diameter. Therefore, our cumulative mass distribution for JF comets is significantly steeper than that derived for small-size, main-belt asteroids. It is also steeper than that derived for near-Earth asteroids, for which Rabinowitz (1993) found an index of -0.66.

It is of special interest to compare our derived mass-distribution (or luminosity function) of JF comets with that derived for the Edgeworth-Kuiper belt population because of its

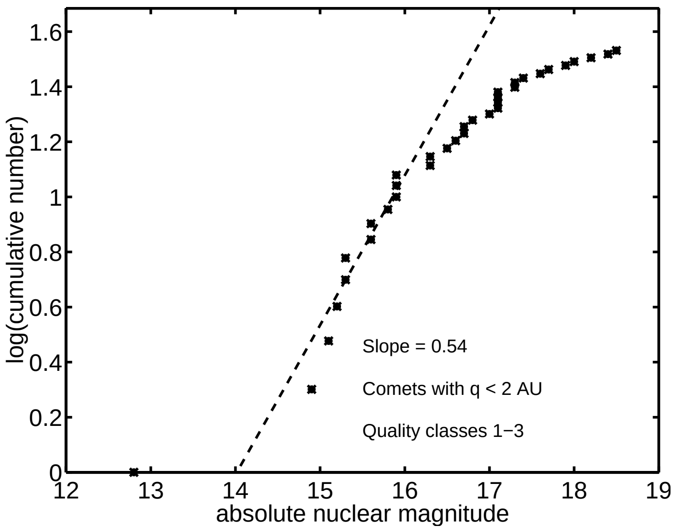

Fig. 4. Cumulative number of JF comets brighter than a given nuclear magnitude H_N. We consider JF comets with $q < 2$ AU and quality classes 1–3.

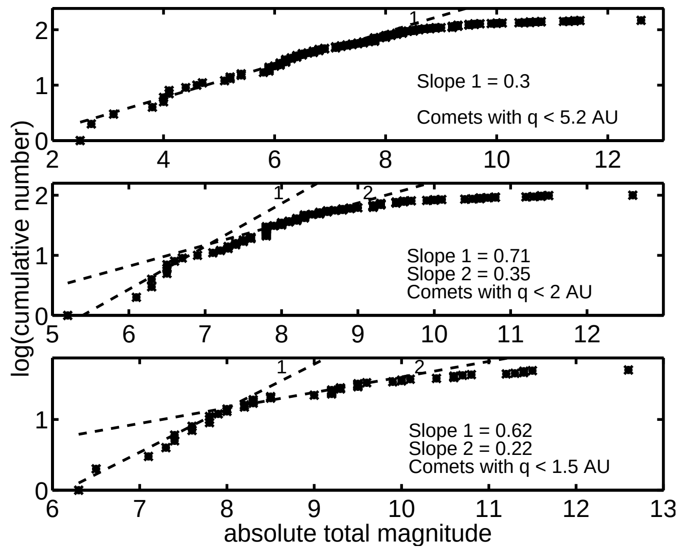

Fig. 5. Cumulative number of JF comets brighter than a given total magnitude H_T for $q < 5.2$ AU, $q < 2$ AU and $q < 1.5$ AU.

importance as a potential source region of JF comets (Fernández 1980). Luu & Jewitt (1998) have presented results of a new survey with the Keck 10-m telescope where they concluded that the magnitude distribution up to red magnitude $m_R = 26.6$ is well fitted by a linear relationship of slope 0.54 ± 0.04. Now, for an assumed low (red) geometric albedo $p_R = 0.04$, this distribution would correspond to objects larger than a few tens km diameter, i.e. significantly larger than most JF comets of our sample. If Luu and Jewitt's distribution of EKb objects can be extrapolated down to JF comet-sized objects, then the close match between both distributions is quite remarkable. Yet, if the Cochran et al. (1995) results from a HST survey are correct (that indicate a high density of EKb objects near the noise limit of the data at $m_R \sim 28.1$), then we will have a significantly steeper cumulative luminosity function (CLF) of small EKb bodies as compared to our sample of JF comets. This is in agreement with the Gladman et al. (1998) result of a CLF of slope 0.76 obtained by combining results from different surveys.

5. The distribution of absolute total magnitudes

We are now able to derive theoretically the expected distribution of absolute total magnitudes $N_T(H_T)$ from the distribution of absolute nuclear magnitudes and the relationships between m_T and m_N given by Eqs. (25) – (27). It is easy to show that if the cumulative distribution of H_N follows the relation $\log N_N(H_N) = C_N + bH_N$, where C_N is a constant and $b = 0.53 \pm 0.05$, the differential distribution of H_N is of the form

$$n_N(H_N)dH_N \propto 10^{bH_N} dH_N \quad (31)$$

From Eqs. (25) and (27) we get the following relations

$$H_N = \frac{1}{1.5} H_T + G_1 \ \ (\mathrm{Low-Active\ Comet}) \quad (32)$$

$$H_N = \frac{1}{0.75} H_T + G_2 \ \ (\mathrm{Active\ Comet}) \quad (33)$$

where G_1 and G_2 are functions of r, Δ and α. We have discarded the relation given by Eq. (26) since it applies to the solar radiation pressure case that, as we have seen (cf. Fig. 2), is only marginal.

If we substitute either Eq. (32) or Eq. (33) into Eq. (31) we get the following distribution functions

$$n_T(H_T)dH_T = F_1 10^{b_1 H_T} dH_T \ \ (\mathrm{Low-Active\ Comet}) \quad (34)$$

$$n_T(H_T)dH_T = F_2 10^{b_2 H_T} dH_T \ \ (\mathrm{Active\ Comet}) \quad (35)$$

where F_1 and F_2 are functions of r, Δ and α, and $b_1 = b/1.5$, $b_2 = b/0.75$.

We note that the resulting distribution of H_T, either differential or cumulative, will be bimodal with slopes

$b_1 \sim 0.35$ (Low–Active Comet)

$b_2 \sim 0.70$ (Active Comet)

The location of the knee will be a function of r, Δ and α. An inspection of Fig. 3 shows that the knee falls at $H_T \sim 10$ for comets with $q \sim 1.5$ AU, and it decreases to $H_T \sim 7$ for $q \sim 2$ AU, and $H_H \sim 5$ for $q \sim 2.5$ AU.

Fig. 5 shows the cumulative distribution of H_T for JF comets with $q < 5.2$ AU, $q < 2$ AU and $q < 1.5$ AU. We can fit a straight line of slope 0.3 for the first plot (representing the cumulative number in a logarithmic scale). This result is in good agreement with that found by Hughes (1988), but it has an important flaw: it puts together comets with small q, whose cumulative distribution $N_T(H_T)$ can be considered to be complete up to $H_T \sim$ 6–7, with distant comets that tend to be bright, and thus to give a spurious overrepresentation of bright comets in the whole population. When we limit the sample to $q < 2$ AU or $q < 1.5$ AU, the picture changes drastically: a bimodal distribution clearly shows up with slopes in good agreement with those given by Eqs. (34) and (35). It is interesting to note that the bimodal character of the cumulative distribution of H_T was already found by Everhart (1967) for the sample of observed long-period comets. From the statistics of comet discovery conditions, Everhart computed a discovery probability function from which he derived the intrinsic distribution of H_T. He also found a bimodal distribution with slopes ~ 0.6 and ~ 0.3 with a change of slope near $H_T \sim 6$.

6. The distribution of perihelion distances and population size

Comet La Hire of 1678 is now recognized as the first observation of a known JF comet, which was identified with an apparition of comet 6P/d'Arrest (Carusi et al. 1991). No new discoveries of JF comets were made until 1766, when D/1766G1 Helfenzrieder was discovered. Since then the discovery rate of JF comets has been steadily increasing, reaching a total number of 166 in 1997 (Marsden & Williams 1997). While until 1892 all the discovered JF comets had $q < 2$ AU, deep sky surveys have led to the discovery of a growing number of distant JF comets, in such a way that about half of the JF comets being discovered nowadays have $q > 2$ AU (Fig. 6).

The number of JF comets that were in Earth-crossing orbits at the moment of their discovery is 14, from which 5 are by now no longer observed (one of them, D/1770L1 Lexell, was scattered into a long-period orbit). In addition, four of these comets have increased their perihelion distances above 1 AU, while only one discovered with $q > 1$ AU has become an Earth-crosser (case of 73P/Schwassmann-Wachmann 3). Only two of the JF comets discovered in the last 50 years had $q < 1$ AU at the moment of their discovery: 1) 103P/Hartley 2, discovered in 1986, with $q = 0.961$ AU. It had a close approach to Jupiter to 0.14 AU in April 1971, before which its perihelion distance was > 1.3 AU (it has now increased again to $q = 1.03$ AU). 2) P/1994P1 (Machholz 2) has $q = 0.75$ AU. Six components were detected of this comet (IAU Circulars 6066, 6070, 6071, 6082 and 6090) which suggests that it was discovered after experiencing a splitting that activated a presumably inert or low-active object (Sekanina 1999). By contrast, 311 Aten-Apollo asteroids ($q < 1.017$ AU) have been discovered during the period 1986–1998.

The above discussion suggests that the population of active JF comets with $q < 1$ AU is near completion, as shown by the very slow growth of this population during the present century (Fig. 6). Leaving aside a few as yet undetected faint members, only the dynamical deflection of a JF comet by Jupiter, or the reactivaction of a dormant comet (Kresák 1987), can provide new members to this group. Taking into consideration a few missing comets, we can place the number of JF comets in Earth-crossing orbits at about 14 with an uncertainty of around ±30%. They can be split into 2 for the interval $0 < q \leq 0.5$ AU, and 12 for $0.5 < q \leq 1$ AU.

Nine JF comets with perihelion distances $1 < q \leq 1.5$ AU have been discovered during the period 1978–1997 (out of a total of 40 discoveries within this range of q). This does not represent a significant increase in the discovery rate as compared

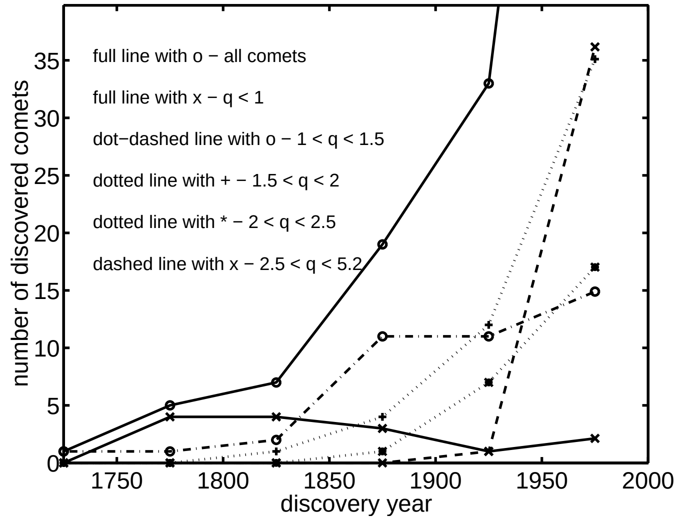

Fig. 6. The discovered number of JF comets per 50-yr interval as a function of the mean discovery year. The whole JF comet sample is plotted and sub-samples for the ranges of q indicated in the figure.

to the last 100 years, despite the efforts devoted to this search. This would suggest that the population of active JF comets with $1 < q \leq 1.5$ AU may not be much larger than the currently known one.

We can make some more quantitative estimate of the population within $1 < q \leq 1.5$ AU from the extrapolation of the population of the brightest comets (assumed to be complete) down to the faintest ones. To this purpose we can make use of the cumulative distribution of absolute magnitudes H_N derived in Sect. 4, namely

$$\log N_N(H_N) = A_N + (0.53 \pm 0.05) \times H_N, \tag{36}$$

where A_N is a constant. As discussed in Sect. 2, no comets fainter than ~ 19.5 are found, and the number rapidly drops for $H_N \gtrsim 18.5$, which corresponds to a nucleus radius of ~ 0.7 km for an assumed geometric albedo $p_v = 0.04$.

We found that the cumulative distribution of H_N, $N_N(H_N)$, is more or less complete down to $H_N \sim 16$ for $q < 2$ AU (cf. Sect. 4). There are four comets with $H_N < 16$ in the range $1 < q \leq 1.5$ AU: Arend-Rigaux, Borrelly, Churyumov-Gerasimenko and Tempel 2. Among them, the last discovery was that of Churyumov-Gerasimenko in 1969, namely, shortly after having its perihelion distance decreased from 2.7 AU to 1.3 AU in 1959. The lack of discoveries in the last 30 years and the fact that the last discovery was of a comet that decreased its q, strengthens the idea that the sample of comets in this range of q brighter than $H_N = 16$ is essentially complete. Therefore, if we extrapolate the observed sample of brighter comets down to $H_N = 18.5$ we get

$$N_N(18.5) = N_N(16) \times 10^{(0.53 \pm 0.05) \times 2.5} \tag{37}$$

which leads to $H_N(18.5) = 85^{+28}_{-22}$, this is about twice the number of discovered JF comets within $1 < q \leq 1.5$ AU, in good agreement with our preliminary conclusion about the high degree of completeness of the observed sample.

The population of active JF comets with $1.5 < q \leq 2$ AU has recorded a moderate increase during the last two decades (1978–1997), as a result of the introduction of new observing programs, rising from 29 to 49 comets (Fig. 6). This is an indication that this sample may still be far from complete. The actual number might be several times greater than the currently known one. As before, let us assume that the population of comets in this range of q is complete for $H_N < 16$. There are 9 comets brighter than $H_N = 16$, from which three were discovered in the last 20 years, though one of them, 116P/Wild 4, was discovered in 1990 after having decreased its perihelion distance from 3.4 AU to 1.99 AU in 1987. As before, if we extrapolate down to $H_N = 18.5$ by means of Eq. (37) we obtain: $N_N(18.5) = 190^{+65}_{-50}$.

To check the previous result, we can use the sample of JF comets with determined absolute total magnitudes (H_T), as given by Kresák & Kresáková (1994). Since total magnitudes are related to the total amount of gas and dust released by the nucleus, they may correlate better with the probability of detection, so it seems appropriate to use them to assess the population size of JF comets, despite the difficulties inherent to the definition of the absolute total magnitude reduced to a standard distance of 1 AU from the Earth and from the Sun. This definition involves the adoption of an index for the slope of the comet's lightcurve, which is usually taken as $n = 4$ (this index has also been adopted by Kresák and Kresáková). According to the results derived in Sect. 5, let us assume that the distribution of absolute total magnitudes of JF comets fainter than $H_T \sim 7$ is well represented – at least up to $H_T \simeq 10.5$ – by

$$\log N_T = A_T + 0.35 \times H_T \tag{38}$$

where N_T is the cumulative number of JF comets brighter than H_T and A_T is a constant. The total magnitude $H_T = 10.5$ roughly corresponds to the nuclear magnitude $H_N = 18.5$ (cf. Fig. 3) and, as before, it may reflect a physical limit beyond which, comets may have much shorter physical lifetimes and tend to be scarce.

For $1.5 \leq q < 2$ AU the cumulative number of JF comets steadily increases with H_T up to $H_T \sim 8.5$, and then it tends to flatten for greater H_T. If we assume that the sample of JF comets in this range of q is essentially complete for $H_T < 8.5$, we can get from Eq. (38) the total population (up to $H_T = 10.5$) as

$$\log \left[\frac{N_T(H_{T_1})}{N_T(H_{T_2})} \right] = 0.35 \times (H_{T_1} - H_{T_2}) \tag{39}$$

where $H_{T_1} = 10.5$ and $H_{T_2} = 8.5$, substituting these values we obtain

$$N_T(10.5) \simeq 5.0 \times N_T(8.5) \tag{40}$$

From the 49 observed JF comets with $1.5 < q \leq 2$ AU, 28 are brighter than $H_T = 8.5$ in Kresák & Kresáková's (1994) catalog. In addition there are 9 comets without estimated total magnitudes and we may presume that some of them have $H_T < 8.5$. Therefore, the number may more properly be about 30. Applying Eq. (40), the number of JF comets brighter than $H_T = 10.5$ is estimated at about 150.

Both results, either with total or nuclear magnitudes, are in reasonable good agreement, and suggest a population of about 150–200 comets, with an uncertainty of a factor of about two either in the sample of comets with $H_N < 16$ or that with $H_T < 8.5$. If we make allowance for some incompleteness in the discovery of bright comets, the population of JF comets will rise somewhat, say to about 250^{+200}_{-100} within the range $1.5 < q \leq 2$ AU.

The population of JF comets in the range $2 < q \leq 2.5$ AU has experienced a significant increase in recent times, doubling its number (from 12 to 24 comets) in the last twenty years (1978–1997). Again, from this single information we can only guess at a much larger population. To try to make some quantitative estimate, let us consider again only the sample of comets brighter than $H_N = 16$ which we may presume to be much closer to completeness. There are seven comets in this group, so an extrapolation up to a magnitude $H_N = 18.5$ will lead to: 150^{+50}_{-40}. We have to make some allowance for some incompleteness in the sample of comets brighter than $H_N = 16$. To this purpose, we note that the current discovery rate of JF comets with $2 < q \leq 2.5$ AU shows a similar slope to that of the discovery rate of JF comets with $1 < q \leq 1.5$ AU during the last century (cf. Fig. 6). The population of JF comets with $1 < q \leq 1.5$ AU has grown by a factor of 2–3 during the present century before getting – seemingly – close to completeness. If we apply the same incompleteness factor of about 2–3, we get a population of about 300–450 comets with an uncertainty of about 50%.

If we consider now total magnitudes, there are 12 comets brighter than $H_T = 7$ in the range $2 < q \leq 2.5$ AU. As before, allowing for undetected fainter JF comets down to $H_T = 10.5$, that number would rise to about 200. If we take instead a more restricted limit of $H_T = 6.5$, the sample decreases to 9 members and the extrapolation down to $H_T = 10.5$ leads to 226 comets, in good agreement with the previous result. The sample of the brightest comets ($H_T < 6.5$) is presumably incomplete (the last two comets of this group were 110P/Hartley 3 and 123P/West-Hartley discovered in 1988 and 1989, respectively), though the slow pace at which comets are discovered in this group could suggest that it may not be much larger. A total magnitude $H_T = 7$ roughly corresponds to a nuclear magnitude $H_N = 16$, so we can adopt the same incompleteness factor of 2–3. Therefore, from the total magnitudes we may estimate the total population in the range $2 < q \leq 2.5$ AU at about 500 comets, with an uncertainty of ± 200 comets.

The results derived from total and nuclear magnitudes are again in fairly good agreement. By combining both we obtain a population of JF comets in the range $2 < q \leq 2.5$ AU of about 450^{+300}_{-200} comets.

There are too few JF comets with $q > 2.5$ AU (only 37) to try to extrapolate the observed population of distant JF comets to its actual size. Nevertheless, from the observation that only the brightest members have so far been discovered, we can advance the hypothesis that they represent only a very tiny fraction of the total population. For instance, there are 6 comets brighter than $H_T = 6$ in the range $2.5 < q \leq 3$ AU, which would lead to about 190 comets down to $H_T = 10.5$. Since the sample

Table 2. Estimated number of JF comets brighter than $H_N = 18.5$ in different q intervals

q (AU)	Number
0.0–0.5	2^{+1}_{-1}
0.5–1.0	12^{+4}_{-4}
1.0–1.5	80^{+40}_{-20}
1.5–2.0	250^{+200}_{-100}
2.0–2.5	450^{+300}_{-200}

of bright comets cannot be considered complete, this should be taken as a lower limit. An additional problem with distant comets is how to extrapolate absolute total magnitudes to a standard $r = 1$ AU when the brightness variation with r strongly varies from comet to comet.

We can make another approach to the problem of the determination of the number of JF comets in the inner planetary region ($q < 5.2$ AU) by considering only those with the brightest absolute nuclear magnitudes. The number of JF comets brighter than $H_N = 15.0$ is 13, from which three were discovered in the last five years: P/1993K2 (Helin-Lawrence), P/1996A1 (Jedicke 2) and P/1997V1 (Larsen), and one re-discovered: P/1998WG22 (Väisälä-Oterma), originally discovered in 1939 and then lost (it is not in our catalog since it was very recently re-discovered by the LINEAR program with an estimated nuclear magnitude $H_N = 13.5$, see IAU Circ. 7064 and MPEC 1998-X19), which suggests that this sample may still be far from complete. There are some distant JF comets without estimated nuclear magnitudes or unreliable ones, as for instance D/1960S1 (van Houten), that may well have $H_N < 15$. Furthermore, we note that there are few bright JF comets with $q > 4$ AU which is very likely due to incompleteness in the discovery of more distant comets. If we just simply take the same number of JF comets per unit of q in the range $4 < q \leq 5.2$ AU as the one observed in the range $3 < q \leq 4$ AU, we would get about 8 comets there. All these considerations sets the minimum number of JF comets brighter than $H_N = 15$ at about 25. Extrapolating to $H_N = 18.5$, we get the total number of JF comets within the inner planetary region ($q < 5.2$ AU) as

$$N_N(18.5) \sim 25 \times 10^{(0.53\pm0.05)\times3.5} = 1800^{+900}_{-600} \, .$$

This estimate should be considered as a lower limit; if we allow for some incompleteness factor in the number of bright comets with $q < 4$ AU, not considered before, the number should rise somewhat. Therefore, the population size of JF comets down to $H_N = 18.5$ may be estimated to be on the order of several thousand comets.

The derived numbers of JF comets within intervals of $\Delta q = 0.5$ AU are shown in Table 2. As a comparison, three theoretical q-distributions for hypothetical comets subject to Jupiter's perturbations are shown in Fig. 7 superimposed on the derived numbers. These curves were derived following the procedure outlined by Fernández (1984) and have been normalized to a total number of 800 comets for $q < 2.5$ AU as derived from

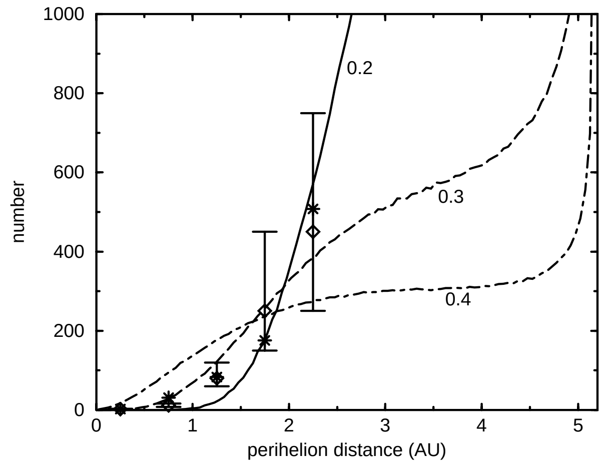

Fig. 7. Estimated number of JF comets within q-intervals of 0.5 AU (diamonds). The curves are for theoretical q-distributions for mean velocities 0.2, 0.3 and 0.4 (in units of Jupiter's orbital velocity, assumed to be circular). The asterisks are values estimated by Levison and Duncan (1997). All the values are normalized to a comet population of 800 for $q < 2.5$ AU.

Table 2. The steep increase in the number of JF comets with q is quite striking. On dynamical grounds, JF comets under the gravitational control of Jupiter exhibit a dropoff toward smaller q that depends on the average encounter velocity with the planet (Fernández 1984). For a typical encounter velocity ~ 0.2–0.3 (in units of Jupiter's circular speed), the expected dropoff is fairly steep since, under such circumstances, it is very difficult for Jupiter to scatter a comet into a small-q orbit. On the other hand, if the encounter velocities were higher ($U \sim 0.4$–0.5), the q-distribution would tend to be much flatter since scattering into small-q orbits becomes much more likely. The encounter velocities U with Jupiter are assumed to have a Maxwellian distribution with mean velocities $\bar{U}$: 0.2, 0.3 and 0.4. We note that U varies very little with the dynamical evolution of a given comet (it should be an invariant in the circular, restricted three-body problem). Periodic comets captured from near-parabolic orbits should have encounter velocities $U > \sqrt{2} - 1$. By contrast, JF comets captured from heliocentric orbits of rather small eccentricities and inclinations would have smaller encounter velocities ($U \simeq 0.1$–0.3). For instance, 29P/Schwassmann-Wachmann 1 has an encounter velocity with Jupiter $U = 0.124$.

Given the large uncertainties involved in the derivation of the current number of JF comets as a function of q, it is difficult to conclude to which particular curve the empirical numbers fit better. It seems that rather high values of U, say $U \gtrsim 0.4$, would give a too flat curve (i.e. too many JF comets with small values of q). Conversely, very low values of U, say $U \lesssim 0.2$, would give a very steep increase in the number of JF comets with q. The best fit seems to be between $\bar{U}$ = 0.2 and $\bar{U}$ = 0.3, at least for $q < 2.5$ AU. Levison & Duncan (1997) derived a q-distribution of fictitious JF comets with $q < 2.5$ AU from numerical simulations of thousands of objects starting in Neptune-encountering orbits and evolving by gravitational encounters with the Jovian planets. Their simulations also assume a certain physical lifetime for the JF comets with $q < 2.5$ AU. Their derived numbers (also normalized to 800 comets) are also shown in Fig. 7. Levison and Duncan's computed q-distribution is somewhat steeper than ours, though given the different procedures, assumptions and uncertainties involved, the discrepancy cannot be taken as very significant.

The previous fit to a curve between $\bar{U}$ = 0.2 and $\bar{U}$ = 0.3 is relevant not only to discussions on possible source regions of JF comets, but also on the total population of JF comets. For a mean encounter velocity of, say $\bar{U} \sim 0.25$, only about 8% of the JF comets crossing Jupiter's orbit will have at a given time $q < 2.5$ AU, i.e. there is a steep increase with q with a concentration close to Jupiter's orbit. Thus, if the number of JF comets with $q < 2.5$ AU is about 800, we can make an extrapolation to set the total number of JF comets crossing Jupiter's orbit ($q \lesssim 5.2$ AU) at about 10^4. This is consistent with what we found before, namely that the JF comet population should be on the order of several thousand members.

7. Concluding remarks

Our knowledge of the physical properties of comet nuclei, in particular those of the Jupiter family, is improving considerably thanks to the intensive use of CCD detectors for early recovery and near-aphelion observations of JF comets. We should add to this the *in situ* observations of the nucleus of 1P/Halley by the Giotto and Vega spacecraft. A picture of a comet nucleus, from one km to several km diameter, of extremely low geometric albedo, very low internal strength and density (e.g. Rickman 1986), and possibly integrated by building blocks (Weissman 1986, Weidenschilling 1994) has emerged. Yet, except for 1P/Halley and a few other well observed JF comets, we are still far from having reliable magnitudes of comet nuclei. Large scatters in most of the observed nuclear magnitudes remain, even for JF comets observed at $r > 3$–4 AU. This scatter may be partially due to some residual activity. In fact, some comets show activity well beyond 10 AU, and 1P/Halley experienced an outburst at 14.3 AU (Sekanina et al. 1992), which suggests that other substances more volatile than H_2O (possibly CO_2 or CO) may be at work. Despite the problems mentioned above, the large body of photometric measurements accumulated during the last few decades, and especially the CCD data taken during the last fifteen years, is of extreme importance to study the physical nature of the comet nucleus and to narrow down the uncertainty range of their magnitudes.

In Paper I we estimated the nuclear magnitudes of 105 comets, which constitutes 64% of the whole observed JF population. From the list of JF comets with derived nuclear magnitudes, 64 have uncertainties within ± 1 mag. Our sample is large enough and the uncertainties – hopefully – small enough to be of value in the study of some statistical properties as, for instance, the luminosity function, the size distribution and population size. Some broad features of this statistical analysis are:

1. the cumulative distribution of absolute nuclear magnitudes follows a power-law of index 0.53 ± 0.05, i.e. steeper than the one derived by Shoemaker & Wolfe (1982) based on Roemer's data;
2. the upper limit for the nuclear magnitude is found to be at ~ 19.5, corresponding to a radius of ~ 0.4 km for an albedo $p_v = 0.04$. The lack of observed JF comets definitely fainter than ~ 19.5 does not rule out their existence, but they seem to be quite scarce suggesting that their physical lifetimes are very short;
3. the total number of JF comets crossing or approaching Jupiter's orbit ($q < 5.2$ AU) up to an absolute nuclear magnitude $H_N = 18.5$ is estimated to be from several thousand to about 10^4. This number is well below the upper limit of $\sim 30,000$ found by Lindgren et al. (1996) based on searches of JF comets in Jupiter's vicinity and on the assumption that half of the comets at Jupiter's distance are inactive and the other half are brightened by 1.5 magnitudes due to activity;
4. a steep increase in the number of JF comets with q is confirmed, which is in good agreement with a population of comets encountering Jupiter at low relative velocities ($U \sim 0.2$–0.3 in terms of Jupiter's orbital velocity). Such low encounter velocities favor an immediate comet source in the Jupiter-Saturn region which itself may be supplied by bodies handed down by the outer planets from the Edgeworth-Kuiper belt and, to some unknown extent, captured by Jupiter from the near-parabolic Oort cloud flux or expelled by collisions and Jovian perturbations from the Trojan clouds (e.g. Marzari et al. 1995).

Much work still has to be done to get better statistics of comet sizes. More measurements of near-aphelion nuclear magnitudes of JF comets will be of fundamental importance for that purpose, allowing us to have the lowest level of coma contamination. Such observations should become possible in a near future using the new generation of telescopes with very large apertures, e.g. Keck or VLT. That will allow us to have available a much larger data set of reliable nuclear magnitudes and advance in some of the preliminary conclusions discussed here.

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