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Pulsation models of δ Scuti variables

II. δ Scuti stars as precise distance indicators

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Abstract. The Hipparcos period-luminosity relation for high-amplitude δ Scuti stars is discussed as an average period-luminosity-colour-metallicity relation comparing the Hipparcos sample of calibrating stars with a more representative sample taken from McNamara (1997). The Hipparcos relation includes systematic effects of both colour and metallicity and agrees with other period-luminosity relations with an accuracy of about ± 0.10 mag.

Direct stellar-evolution and pulsation modelling is used to study effects on the average period-luminosity relation from the location of the high-amplitude variables within the instability strip given as a range in effective temperature (colour term) and the metallicity given as [Fe/H]. Using accurate data for the instability strip given by McNamara precise agreement with the Hipparcos relation is found. By fitting the bolometric magnitude as a function of pulsation period, effective temperature and metallicity an accuracy of 0.02–0.03 mag is obtained both within the narrow high-amplitude strip and including a much broader strip covering most of the full δ Sct instability strip. Problems in practical application of this relation are briefly considered. Using $uvby\beta$ narrow-band photometry it seems possible to obtain this accuracy in determinations of distances to e.g. the Galactic Bulge and the Magellanic Clouds.

Key words: stars: distances – stars: fundamental parameters – stars: oscillations – stars: variables: δ Sct

1. Introduction

Primary distance indicators are classical Cepheids, RR Lyrae variables and supernovae. Hipparcos data have given impressive progress in several areas, but unexplained discrepancies are still present (e.g. Turon & Perryman 1999; Pont 1999). New independent checks are clearly needed. In the present paper we discuss the distance scale defined by high-amplitude δ Scuti stars (HADS in the following, conventionally defined by the requirement that the amplitude i V, A_V, is larger than 0.3 mag) and show that they define a distance scale with an accuracy of about ± 0.10 mag in simple period-luminosity (PL) relations and ± 0.03 mag in relations including colour and metallicity terms. Distances based on HADS can be determined for several globular clusters, the Galactic bulge and the Magellanic Clouds.

In Sect. 2 we discuss the properties of the Hipparcos PL relation (for short PLR in the following), comparing with a sample of δ Sct stars discussed in detail by McNamara (1997). In Sect. 3 we use theoretical evolution tracks to derive improved PLR and period-luminosity-colour-metallicity relations (PLCZR) for HADS, and Sect. 4 contains an extension of the fitting formulae to include the full width of the instability strip. Finally, in Sect. 5 we give a short discussion emphasizing application of $uvby\beta$ narrow-band photometry for precise distance determination using δ Sct stars, and summarize our conclusions.

2. Properties of the Hipparcos PLR

The standard PLR form is

$$M_V = A \log \Pi_0 + B \,, \tag{1}$$

where the coefficients A and B are determined from a set of observed $(\Pi_0,\ M_V)$ for a sample of stars. Π_0 is the fundamental mode period, and we always use periods in unit of [d]. Fundamental values of the absolute V magnitude M_V are based on trigonometric parallaxes. For HADS Hipparcos provided the first data accurate enough to derive a fundamental PLR. Most earlier PLRs for HADS were based on M_V-values taken from photometric calibrations derived for "normal" low-amplitude δ Sct stars. Petersen & Høg (1998) compared the Hipparcos PLR with these earlier relations and concluded that all deviations larger than about 0.1 mag between the Hipparcos PLR and earlier results can be explained by uncertainties in the data available before the Hipparcos Catalogue.

For a more detailed discussion of errors in PLRs it is useful to rewrite Eq. (1):

$$M_V = a \log (\Pi_0/\Pi_{\rm r}) + \tilde{b} \,, \tag{2}$$

where $\Pi_{\rm r}$ is a conveniently chosen reference period and a and $\tilde{b}$ again are parameters to be determined by the Least-Squares (LSQ) solution for a known sample. Choosing $\Pi_{\rm r}$ near the middle of the abscissa range of our data, we obtain the advantages

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Table 1. Coefficients $A = a, B$ and $\tilde{b}$, and the scatter σ (all given in mag) in Eqs. (1) and (2) with $\Pi_{\rm r} = 0.1$ d for simple PL relations. Data for Hipparcos PLR are given both with and without Lutz-Kelker corrections (LKC), see text for details

PL relation	$A = a$	B	$\tilde{b}$	σ
Hipparcos, no LKC	-3.73	-1.90	1.83	0.10
M_V	±0.57	±0.59	±0.10	
Hipparcos with LKC	-4.096	-2.340	1.756	0.09
M_V	±0.495	±0.513	±0.088	
McNamara (1997)	-3.725	-1.990	1.735	0.10
26 stars, M_V	±0.089	±0.087	±0.021	
McNamara (1997)	-3.597		1.629	0.09
26 stars, $M_{\rm bol}$	±0.080		±0.019	

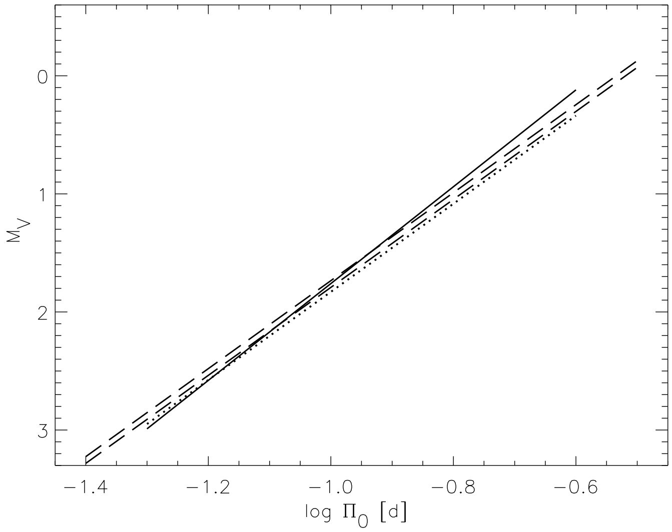

Fig. 1. Comparison of Hipparcos PL relations (full line: including Lutz-Kelker corrections, dotted line: no L-K corrections) with McNamara's solutions (dashed lines, the lower one with adjusted zero point). See text for details

that covariances may be neglected and the standard error of M_V (or the bolometric magnitude $M_{\rm bol}$) predicted from Eq. (2) is approximately equal to $\sigma_{\tilde{b}}$ at typical periods near the middle of the abscissa range. An estimate of the accuracy of $M_V/M_{\rm bol}$ calculated by PLRs from the observed period of a star is given by the formal scatter (standard deviation) of the LSQ fitting (see Petersen & Høg 1998 for details). For the Hipparcos HADS sample and the comparison sample from McNamara (1997) the optimal choice is $\Pi_{\rm r} = 0.1$ d. In Sect. 3 we construct improved PLCZR for the Pop. I stars of relatively long oscillation periods. Then we use $\log \Pi_{\rm r} = -0.85$, $\Pi_{\rm r} = 0.141$ d. This gives a difference in the constant term of $\simeq 0.7$ mag between the two cases.

Table 1 gives detailed information on the simple PLRs we concentrate on here. The two Hipparcos PLRs are based on six stars, of which only three have high statistical weights. Evidently, this is a weak point which we will consider in some detail in the following. For an explanation of the Lutz-Kelker correction we refer to Petersen & Høg (1998). Here we only mention that this correction takes an observational bias into account, and therefore the PLR including the Lutz-Kelker correction is the correct one in a statistical sense. We emphasize that for distance determinations all differences between the PLRs in Table 1 are insignificant. It is seen that all three relations have $\sigma_{M_V} \simeq \pm0.10$ mag at typical periods. For the Hipparcos relations the formal σ_{M_V} increases significantly for longer/shorter periods, reaching $\simeq \pm0.20$ mag at the end-points of the relevant interval.

McNamara (1997) used accurate and homogeneous $uvby\beta$ photometry to determine mean temperature and metal abundance for 26 HADS. Next, for each star the mean effective temperature $T_{\rm eff}$ and the accurately know pulsation period were used to interpolate between evolutionary tracks for various masses in the (log Π_0, log $T_{\rm eff}$) calibration diagram calculated for the relevant metallicity, giving the mass and $M_{\rm bol}$. Finally, he determined $M_V = M_{\rm bol} - BC$, BC being the bolometric correction. McNamara's LSQ solution is reproduced in Table 1 and compared with the Hipparcos solutions in Fig. 1. In Table 1 we also give the LSQ solution for McNamara's $M_{\rm bol}$ values. For HADS the average bolometric correction from McNamara's data becomes

$$BC = 0.128 \log \Pi_0 + 0.022 \; . \tag{3}$$

The systematic change from -0.07 mag at $\Pi_0 \simeq 0.2$ d for typical Population I δ Sct stars to -0.14 mag at $\Pi_0 \simeq 0.05$ d for Population II stars reflects the systematic change in metallicity for the McNamara sample. At Π_0 = 0.1 d BC becomes -0.106, giving a best Hipparcos estimate of $M_{\rm bol}$ of 1.650 in good agreement with McNamara's result.

McNamara preferred to adjust the zero point of his relation to the Hipparcos scale (without Lutz-Kelker corrections). However, the resulting correction of only 0.057 is clearly well within the uncertainties of these PLRs. For distance determinations using HADS we recommend

$$M_V = -3.725 \log \Pi_0 - 1.969 \; , \tag{4}$$

with a zero point adjusted to the Hipparcos value at the period $\Pi_0 = 0.1$ d (including Lutz-Kelker corrections, $\tilde{b} = 1.756$). This is 0.036 mag brighter than preferred by McNamara in his Eq. (4). The accuracy of an M_V-value given by Eq. (4) is approximately ±0.10 mag in the whole period interval.

Eq. (4) is based on the sample of stars (26) included in McNamara's Table 2. His Fig. 1 shows a systematic increase in [Fe/H] with increasing period. Thus *Eq. (4) gives an average PLR where this systematic variation in metallicity for HADS field stars is taken into account*. It is also important that the globular-cluster stars included in this figure follow the log Π_0 - [Fe/H] relation defined by the field stars. This fact indicates that Eq. (4) may also be valid for globular-cluster HADS.

We emphasize that the two stars of particularly high weight in the Hipparcos PLR, AI Vel and SX Phe, are located close to the mean curve in McNamara's Fig. 1. This may give an understanding of why the Hipparcos PLR is in perfect agreement with

McNamara's PLR. Since AI Vel and SX Phe are typical examples of Population I and II, respectively, the systematic effect of varying [Fe/H] with period is automatically included in Hipparcos LSQ solutions. (In fact, the probability of finding a HADS so metal poor as SX Phe and so close is very small.) Had no Pop. II star been included in the sample available for derivation of the Hipparcos PLR an [Fe/H] effect could obviously not have been taken into account. We consider [Fe/H] effects in more detail in the next section using modelling.

Deriving accurate positions in the HR diagram McNamara (1997) showed that the width of the HADS instability strip is only $\simeq$ 300 K. Comparing e.g. with Fig. 1 of Breger (1990) it is seen that this corresponds to only 20–25% of the full observed width of the δ Sct instability strip. McNamara remarked that if allowance is made for observational errors the HADS strip width may be only $\simeq$ 200 K. From his Fig. 2 it is seen that the luminous, long-period HADS of Pop. I have systematically lower effective temperatures than the short-period Pop. II variables. Thus the straightforward Hipparcos PLR includes also this variation. The fact that HADS occur only in a very narrow region in the HR diagram is important for the accuracy of the HADS PLR as we discuss in detail in the next section. An immediate consequence is that a colour-correction term in the PLR must be relatively small. This fact was used by Petersen (1999) to give a period-luminosity-metallicity relation.

While the Hipparcos PLR is purely empirical, McNamara's PLR is based on absolute magnitudes determined by means of detailed stellar-evolution models. Therefore, the agreement between these PLRs gives a confirmation of the model sequences used by McNamara. This confirmation of the theoretical models is not unexpected. To our knowledge, all new detailed tests of stellar models in the lower main-sequence by means of the Hipparcos parallaxes have resulted in confirmation of standard models with no or "moderate" convective core overshooting. In particular, several asteroseismological studies have provided convincing results. Paper I of this series (Petersen & Christensen-Dalsgaard 1996) predicted parallaxes of SX Phe and AI Vel, which were confirmed by the accurate Hipparcos data (Høg & Petersen 1997). Further examples are e.g. Bedding et al. (1998) and Viskum et al. (1998).

3. Improved PL and PLCZ relations for HADS

Theoretical modelling of the Cepheid PL relation has a long history (e.g. Stothers 1988; Chiosi et al. 1993; Sasselov et al. 1997; Baraffe et al. 1998) and is important both for an improved understanding of the physics behind the PLR and for deriving theoretical PLRs as checks of the empirical relations. In order to improve the accuracy, period-luminosity-colour relations (PLCR in the following) are often used. In recent years the emphasis has been on determination of the correction term to PLRs for changes in metal content, Z or [Fe/H], (e.g. Feast & Catchpole 1997; Baraffe et al. 1998).

From stellar-evolution and pulsation theory it is well known that the basic relation must be a period-luminosity-colour-metal relation (PLCZR) rather than a simple unambiguous PLR. This problem has been discussed in great detail for classical Cepheids (e.g. Stothers 1988; Beaulieu et al. 1997; Sasselov et al. 1997).

In the present study we focus on Pop. I δ Sct stars. We derive accurate theoretical relations based on stellar-model sequences with the physics described by Petersen & Christensen-Dalsgaard (1996), and compare with the sample given by McNamara (1997). Of the 26 HADS in McNamara's sample 22 have [Fe/H] ≥ -0.65. Here we regard these stars as Pop. I. Pop. II is represented by only four stars, two with [Fe/H] $= -1.4$ and two with [Fe/H] $= -2.4$. This is a small sample for the definition of the Pop. II instability strip in the HR diagram, and determination of $T_{\rm eff}$ and BC becomes more uncertain for extreme Pop. II stars. In the following analysis we include only the 22 stars of Pop. I, which we assume to define a representative sample.

We have studied relations for both M_V and $M_{\rm bol}$. In McNamara's analysis $M_{\rm bol}$ is the primary quantity, while M_V is determined using BC. Noting that this is also the case for all theoretically derived PLRs we choose to analyse $M_{\rm bol}$ rather than M_V. This gives two advantages: (i) we obtain a slightly improved accuracy in the calibrating formulae, and (ii) our formulae do not refer to a specific observational system. The disadvantage is of course that now our formulae cannot be applied directly to e.g. $BVRI$-data in a very simple way corresponding to the analysis in Baraffe et al. (1998). In order to improve the accuracy we suggest a somewhat more involved application of $uvby\beta$-photometry. In the discussion we estimate the uncertainty of this procedure to be 0.02–0.03 mag in the distance modulus.

3.1. The McNamara sample of 22 Population I stars

In order to decrease the effect of covariances in PLCZR we choose as variables in our LSQ solutions quantities that are zero in the middle of the relevant parameter intervals. We introduce

$$p = \log \Pi_0 + 0.85 , \quad t = \log T_{\rm eff} - 3.866 , \qquad (5)$$
$$f = [\mathrm{Fe/H}] + 0.20 ,$$

as period, colour and metallicity variable, respectively, and calculate LSQ fits for the coefficients $a,\ b,\ c$ and d in

$$M_{\rm bol} = a\,p + b + c\,t + d\,f\ . \qquad (6)$$

This gives a reference period $\Pi_{\rm r} = 0.1413$ d optimal for Pop. I HADS. Thus our PLCZR is a $M_{\rm bol} - \log \Pi_0 - \log T_{\rm eff} -$ [Fe/H] relation. To gauge the significance of the different terms, we also consider fits where only some of the terms are included. In Table 2 main properties of four such fits are given in the upper half of the table; entries left blank correspond to terms not included in the corresponding fit.

McNamara (1997) briefly discussed the possibility of reducing the error in M_V calculated from Eq. (4) by introducing more than the $\log \Pi_0$ term, in particular a [Fe/H] term. He found slightly higher luminosities at the same period for the metal-strong stars than for the metal-poor stars. This is in agreement with the tendency (not statistically significant) in Table 2 with $d = \partial M_{\rm bol}/\partial\,[\mathrm{Fe/H}] = -0.070 \pm 0.069$. In contrast, we find a

Table 2. Comparison of PLCZR for McNamara's sample of 22 HADS (upper half of table) with the Pop. I sample of 122 theoretical models shown in Fig. 2 (lower half). Coefficients $a,\ b,\ c$ and d, and the scatter σ [all in mag] in Eq. (6) are given for fits to $M_{\rm bol}$. In each section the four lines correspond to fits involving Π_0, Π_0 and $T_{\rm eff}$, Π_0 and [Fe/H], and Π_0, $T_{\rm eff}$ and [Fe/H], respectively. For comparison with Table 1, b(0.1 d) is $M_{\rm bol}$ at the period 0.1 d ($p = -0.15$) calculated for $t = f = 0$

a	b	c	d	σ	b(0.1 d)
-3.469	1.078			0.075	1.598
±0.093	±0.016				
-4.037	1.088	-9.00		0.053	1.694
±0.147	±0.012	±2.07			
-3.387	1.081		-0.070	0.073	1.589
±0.124	±0.017		±0.069		
-4.000	1.099	-11.72	-0.177	0.035	1.699
±0.101	±0.009	±1.53	±0.037		
a	b	c	d	σ	b(0.1 d)
-3.292	1.112			0.076	1.606
±0.040	±0.008				
-3.744	1.106	-8.80		0.061	1.667
±0.064	±0.007	±1.08			
-3.206	1.109		-0.186	0.065	1.590
±0.036	±0.007		±0.028		
-3.785	1.099	-12.06	-0.271	0.026	1.667
±0.028	±0.003	±0.49	±0.012		

significant colour term: $c = \partial M_{\rm bol}/\partial \log T_{\rm eff} = -9.00 \pm 2.07$. And more remarkably, when we introduce *both* colour and metallicity — which according to basic theory is necessary to give a physically correct description of the problem — we obtain a considerably improved PLCZR. Note that the formal scatter σ, estimating the uncertainty expected from one observation with negligible errors, decreases from 0.075 mag in the simple PLR to 0.035 mag in the PLCZR. We conclude that observations of HADS can give very precise distance moduli.

The coefficient $c = -11.7$ of the $T_{\rm eff}$ term in Eq. (6) may seem alarming. However, within the HADS strip the maximum distance from the middle line is about 150 K, i.e., $\Delta \log T_{\rm eff} \leq 0.009$. The corresponding colour correction is 0.10 mag with a formal uncertainty of 0.01 mag. Note also that this term is not primarily connected with evolutionary changes in the stars along evolution tracks: it is dominated by $\partial M_{\rm bol}/\partial \log T_{\rm eff}$ for constant Π_0, i.e., approximately constant radius, which is -10.

Within Pop. I we expect a spread Δ [Fe/H] of up to about ±0.3; hence, according to the PLCZR, we obtain a variation $\Delta\, M_{\rm bol}$ of up to $\mp0.05 \pm 0.01$ mag from differences in metallicity.

3.2. Theoretical modelling of PLR and PLCZR for HADS

We now discuss theoretical modelling of PLCZR for HADS. Baraffe et al. (1998) gave an attractive derivation of PLR for classical Cepheids based on self-consistent stellar-evolution and pulsation calculations, and showed that their data are in good agreement with the observations by the MACHO and EROS collaborations of Cepheids in the Magellanic Clouds. In the following we shall use almost the same methods for Pop. I δ Sct stars, except for three important modifications that are necessary for δ Sct stars.

While mode identification is usually not considered as a serious problem for classical Cepheids, in δ Sct stars — even in the stars oscillating in one dominant mode — it is not known in most cases precisely which mode has been observed, in the complicated normal-mode spectrum. Therefore, at present only the small group of HADS stars can be used. Most HADS oscillate in the fundamental mode, and often two modes are observed giving a safe mode identification (e.g. McNamara 1997; Petersen & Høg 1998). In other cases with safe mode identification (at present very few for "normal" low-amplitude δ Sct variables) a good model will usually be available, and either the fundamental mode is among the observed oscillations or its period can be estimated and used in PLCZR.

The second modification is necessary because at present the HADS instability strip in the HR diagram cannot be predicted theoretically; in order to select the oscillating models along an evolution track the relevant instability region must be known. Here we simply choose the precisely located and narrow empirical instability strip determined by McNamara (1997) and briefly described above.

The third modification is due to the fact that HADS are not observed close to the $ZAMS$. In the following we exclude all models situated less than 0.8 mag above the relevant $ZAMS$ from the LSQ solutions. The choice of 0.8 mag as the limiting value is rather arbitrary. However, tests using considerably lower or higher values result in most cases in only small changes in the resulting LSQ solutions.

Fig. 2 illustrates our modelling of the McNamara sample of HADS. We cover the relevant part of the HR diagram with standard evolution tracks of metal content $Z = 0.005, 0.01$ and 0.02, and use the selection criteria just mentioned. Individual models within McNamara's HADS instability strip are marked by symbols. Fig. 3 gives the corresponding simple PL diagram. The full line is the LSQ fit to the 122 models selected, while the dashed line is the Hipparcos PLR including Lutz-Kelker corrections transformed to $M_{\rm bol}$ using Eq. (3) for BC. Within the expected uncertainty of the Hipparcos PRL of ±0.10 mag good agreement is seen. For transformation of the model parameters involved we use $T_{\rm eff,\odot} = 5785$ K, $M_{\rm bol,\odot} = 4.72$ and $Z_\odot$ = 0.019.

Is this an adequate modelling of the observed sample? We first note that at the longest observed periods, $\log \Pi_0 \simeq -0.6$, we have only metal-rich stars ($Z \simeq 0.02$) in both cases. For $Z \simeq 0.005$ we have chosen the highest model mass $M = 1.8\,M_\odot$ This results in a few metal-poor models with $\log \Pi_0 \simeq -0.8$, and an increasing number of such models in the strip for lower periods. It is interesting that our selection criterion requiring a position at least 0.8 mag above $ZAMS$ in the HR diagram results in a short-period limit $\log \Pi_0 \simeq -1.0$ for $Z \simeq 0.02$

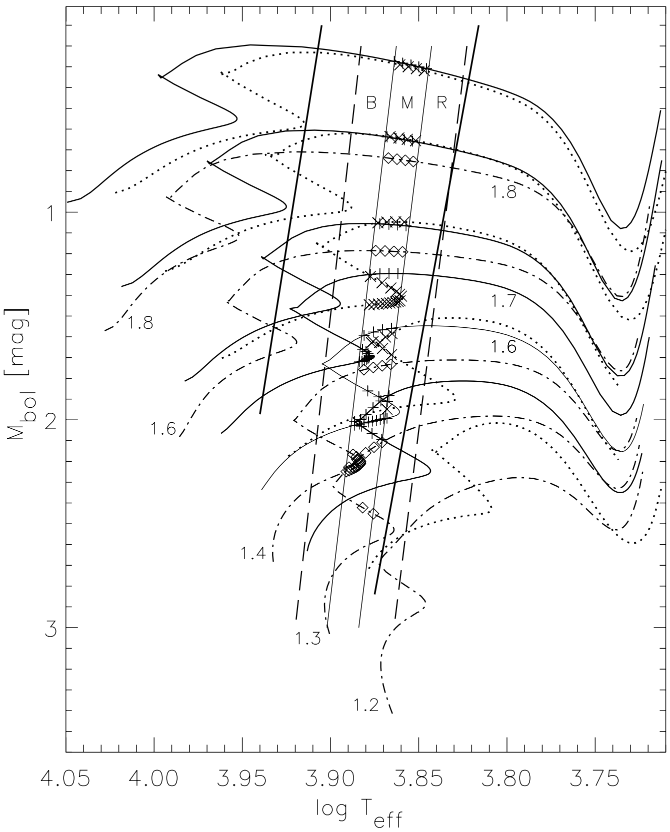

Fig. 2. HR diagram showing standard evolution tracks calculated for $Z = 0.005$ (dash-dotted curves), 0.01 (full curves) and $Z = 0.02$ (dotted curves). Masses are 1.2, 1.3, 1.4, 1.6 and 1.8 $M_\odot$ (all marked) for $Z = 0.005$, 1.5, 1.6 (marked), 1.7 (marked), 2.0 and 2.2 $M_\odot$ for $Z = 0.01$, and 1.6, 1.8, 2.0, 2.2 and 2.4 $M_\odot$ for $Z = 0.02$. Each model within the HADS instability strip (marked M and delimited by thin full lines) is given by a symbol. Adjacent strips B and R are included in some PLCZR; see text for details. Fat lines indicate the limits of the full Pop. I δ Sct instability strip (Breger 1990)

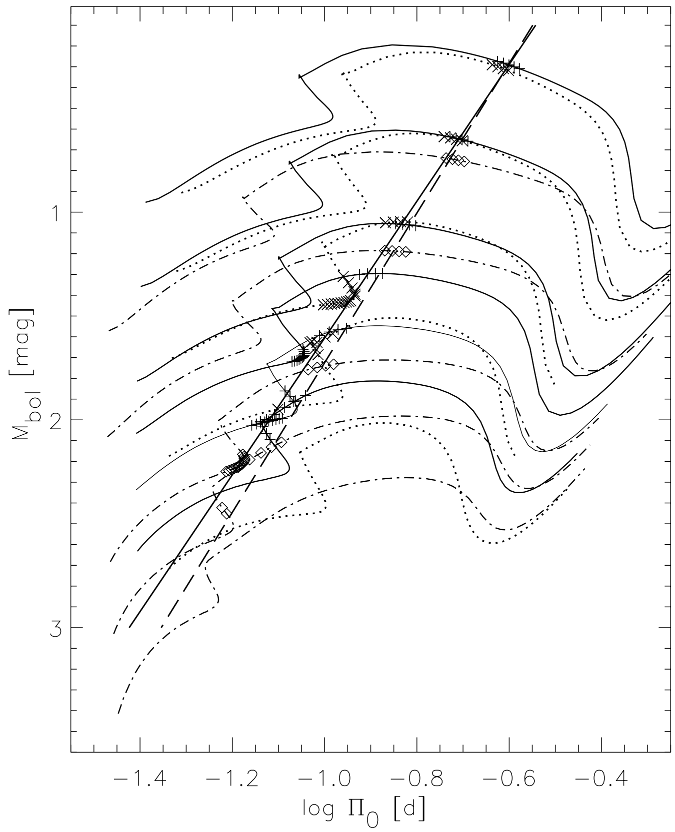

Fig. 3. Comparison of simple PL relations with the standard evolution tracks from Fig. 2 here shown with the same coding. Each model within the HADS instability strip is marked by a symbol. The full line is the LSQ solution for the 122 marked models. The dashed line gives the Hipparcos solution. These solutions are in good agreement within the expected uncertainty

and $\log \Pi_0 \simeq -1.2$ for $Z \simeq 0.02$, precisely as observed. If we had included all MS models within the strip, this limit would become $\simeq -1.3$ for $Z \simeq 0.02$ and $\simeq -1.5$ for $Z \simeq 0.005$, much lower than observed.

In the lower half of Table 2 we give LSQ fits for the 122 HADS models calculated precisely as the fits for the McNamara sample. A remarkable agreement between the two samples is seen. An important quantity is the zero-point b giving the mean $M_{\rm bol}$ at the midpoint of the period interval for average $\log T_{\rm eff}$ and [Fe/H]. For the eight fits in Table 2 the maximum deviation from the mean $b = 1.096$ mag is 0.018 mag. The other coefficients in the fits to the observed and theoretical sample are also seen to agree very well. The formal errors are a factor of about 3 smaller in the fits to models than for the observations. In fact, there are obviously no significant random errors associated with the computed frequencies; thus the deviations from the relation (6), contributing to σ, come from the systematic residuals of the computed results from this simple linear expression. Furthermore, the theoretical fit is based on 122 models, in contrast to the only 22 observed stars. The scatter σ varies in almost the same way in the two cases, decreasing significantly to 0.035 mag in the observed stars and 0.026 mag in the model sample, when all terms in the PLCZR are included. This fact strongly indicates that our modelling is adequate, and that it is possible to use HADS for very precise distance determination with an uncertainty in the distance modulus of 0.026 mag for one observation with negligible errors.

4. The full δ Sct instability strip

While the width in $\log T_{\rm eff}$ of the HADS instability strip is about 0.02, the observed width of the full δ Sct instability strip is much larger, around 0.08 (e.g. Breger 1990, Fig. 2). This leads to larger scatter in PLR including models in the whole strip than in the HADS relations considered until now. In the region R (cf. Fig. 2) adjacent to but redder than McNamara's HADS region M, for the same $M_{\rm bol}$ the periods are larger than in M, whereas in the bluer region B the periods are shorter. This is one reason for the large scatter in observed PL diagrams for δ Sct stars (e.g. Antonello & Mantegazza 1997). Another and more important reason is that most δ Sct stars oscillate in high-order p modes with periods much shorter than Π_0. Only in rare cases with safe mode identification such stars can be used for distance determination.

Table 3 compares LSQ fits for the B, M and R regions of the instability strip with fits for broader substrips. The upper part of the table gives simple PLR. The scatter, σ, estimates the average deviation of a model from the mean PLR. As expected, σ is relatively small, around 0.08 mag, for the narrow substrips, increases (to $\simeq 0.13$ mag) for strips of width $\Delta \log T_{\rm eff} = 0.04$ and is largest, 0.19 mag, for the sample of 332 models in the combined BMR region. Our key parameter b is seen to vary ± 0.20 mag from case M to B and R.

The second part of Table 3 gives PLCZR for the same six samples of models. Here we find that σ in all cases is around 0.025 mag. Thus we conclude that if colour and metallicity terms can be taken properly into account very precise distance determinations are possible in the whole δ Sct instability strip. In the discussion we comment on interesting targets for such studies and suitable observational procedures.

The last section of Table 3 compares the influence from colour and metallicity corrections when the full strip is considered. It is seen that the colour term is by far the most important, in agreement with the indication from McNamara's sample of 22 well-observed HADS. While introduction of the [Fe/H] term gives a small reduction of σ from 0.190 mag to 0.176 mag, the $\log T_{\rm eff}$ term results in a reduction to 0.063 mag.

Table 3. PLR and PLCZR for six choices of instability strip. Cases B (blue substrip), M (middle) and R (red) have a width in $\log T_{\rm eff}$ of 0.02. The combined cases BM and MR include two regions and BMR covers most of the strip (cf. Fig. 2 and text). Column 1 gives the choice of strip and in line 2 for each LSQ solution the number of models included. Coefficients a, b, c and d, and the scatter σ [all in mag] are given precisely as in Table 2

Strip	a	b	c	d	σ
M	−3.292	1.112			0.076
122	±0.040	±0.008			
R	−3.317	1.318			0.083
117	±0.038	±0.008			
B	−3.378	0.878			0.093
93	±0.064	±0.013			
MR	−3.260	1.218			0.131
239	±0.045	±0.010			
BM	−3.316	1.013			0.139
215	±0.058	±0.012			
BMR	−3.244	1.129			0.190
332	±0.058	±0.012			
M	−3.785	1.099	−12.06	−0.271	0.026
122	±0.028	±0.003	±0.49	±0.012	
R	−3.806	1.093	−12.52	−0.272	0.025
117	±0.028	±0.009	±0.52	±0.011	
B	−3.761	1.099	−12.08	−0.294	0.023
93	±0.028	±0.009	±0.48	±0.012	
MR	−3.790	1.099	−12.18	−0.271	0.026
239	±0.013	±0.003	±0.18	±0.008	
BM	−3.781	1.100	−12.18	−0.281	0.025
215	±0.014	±0.002	±0.17	±0.008	
BMR	−3.785	1.099	−12.18	−0.277	0.025
332	±0.010	±0.002	±0.10	±0.007	
BMR	−3.924	1.103	−12.50		0.063
332	±0.023	±0.004	±0.24		
BMR	−3.093	1.123		−0.343	0.176
332	±0.057	±0.011		±0.046	

5. Discussion and conclusions

The Hipparcos PL relations reproduced in Table 1 are the only fundamental PLR for high-amplitude δ Sct stars directly derived through trigonometrical parallaxes. The relation including Lutz-Kelker corrections is the correct one according to statistical theory. We emphasize that except for the longest relevant periods ($\Pi_0 \geq 0.2$ d), where no high-quality Hipparcos parallax is available, the difference between the two Hipparcos PLRs is insignificant.

From stellar-evolution and pulsation theory it is well known that the basic relation must be a period-luminosity-colour-metal relation (PLCZR) rather than a simple unambiguous PLR. Therefore, the Hipparcos PLR must include systematic variations in colour and metallicity with period. Since the Hipparcos PLR is based essentially on only three stars of high statistical weight, it is not evident that this relation gives a representative average PLR for all HADS with the accuracy indicated by the formal statistical uncertainties.

McNamara (1997) obtained a totally independent PLR for HADS in very good agreement with the Hipparcos PLR, as discussed in Sect. 1. Inspecting his data, it is seen that the two stars of highest weight in the Hipparcos solution, AI Vel and SX Phe, seem to be typical examples of Population I and II, respectively. This may give an understanding of why the Hipparcos PLR gives a good average PLR for field HADS.

Another important point is McNamara's discovery of the fact that the width of the HADS instability strip at constant luminosity is very small, at most $\simeq 300$ K. This is only 20–25% of the total width of the δ Sct instability strip. Therefore the colour term in the PLCZR for HADS becomes much less important than for a PLCZR for all δ Sct stars (assuming safe mode identification allowing derivation of Π_0). Thus the PLCZR for HADS should reduce to a P-L-[Fe/H] relation. McNamara briefly considered this possibility, but did not solve for this solution, which is problematic due to a strong correlation between period and [Fe/H] in his relatively small sample of stars. We here give such PLZR in Table 2. However, we find that the colour term given by the coefficient c in PLCZR is more important than the metallicity term (d) both in the observed sample and in the theoretical simulation.

For classical Cepheids the width of the instability strip is around 1400 K. This is at least four times wider than the HADS strip. Similarly, the interval in $\log \Pi_0$ for same $M_V/M_{\rm bol}$ within the instability strip is about 0.4 for classical Cepheids, which is also about four times the width in our Fig. 3. This fact explains why we find relatively small scatter in the HADS relations, compared with the Cepheid case.

Table 2 shows for HADS a considerable decrease in the scatter σ by introduction of colour and metallicity terms, as expected from basic theory. However, the resulting σ of 0.026 mag is remarkably small and indicates that HADS can be used for very precise distance determinations. Perhaps more surprising, the second part of Table 3 shows that even including all models in a broad instability strip the resulting PLCZR has $\sigma \simeq 0.025$ mag. The nearly constant coefficients in the second part of Table 3, in particular the values of b ($M_{\rm bol}$ at $\log \Pi_0 = -0.85$ and average $\log T_{\rm eff}$ and [Fe/H]) show that in practical applications the whole δ Sct instability strip can be covered by the same formula (BMR case).

We recall that correct mode identification is required for application of this PLCZR. For HADS the arguments for radial oscillations in the fundamental mode and the first overtone are strong (although there is no definitive proof). The observed period ratios of AI Vel and SX Phe are in precise agreement with theoretical predictions for the observed metal content (Petersen & Christensen-Dalsgaard 1996). If a nonradial mode of degree $\ell = 1$ or 2 were excited, the period ratio should be somewhat different. Rodríguez et al. (1995) used all available multicolour data in the Strömgren and Johnson photometric systems for 28 HADS and three variables of amplitude 0.09–0.20 mag to analyse for the type of pulsation on the basis of the phase shifts and amplitude ratios between observed light and colour variations. Their results indicate that all these stars are radial pulsators. Such direct methods for distinguishing between radial and nonradial modes are improving in sensitivity (e.g. Viskum et al. 1998). In future applications they can be expected to give reliable results for HADS. As far as we know, no claim has been made until now for the presence of nonradial modes in HADS.

As a test case for a low-amplitude multi-mode variable star we take FG Virginis studied in detail by Breger et al. (1999) and Viskum et al. (1998). FG Vir has a complicated mode spectrum and a fundamental mode of low amplitude with period 0.0822 d. From the parameters based on asteroseismological density $\log T_{\rm eff} = 3.876 \pm 0.007$ and $M_{\rm bol} = 1.848 \pm 0.070$, implying a position in the HADS region M. These data are in agreement with the somewhat less accurate Hipparcos results. Unfortunately, an accurate [Fe/H] is not available; we have to neglect the metallicity correction here. Using Table 3 our simple PLR gives $M_{\rm bol} = 1.89 \pm 0.08$ and the PLCZR gives $M_{\rm bol} = 1.873 \pm 0.026$ for case M and $M_{\rm bol} = 1.872 \pm 0.025$ for case BMR, in good agreement with the asteroseismological analysis. More precise tests could be obtained from similar stars with precise [Fe/H] and Hipparcos parallaxes.

5.1. Accurate distance determinations

Important steps in the cosmic distance scale are the Galactic Bulge and the Magellanic Clouds. As discussed in many recent papers there are at present still discrepancies between different results based on classical Cepheids, RR Lyrae variables and other distance indicators. What is required for an improved distance determination from δ Sct stars?

Provided that the HADS in the Galactic Bulge and LMC are similar to those in the solar neighbourhood they satisfy the average PLR in Eq. (4), and we see no reason for significant differences. Even if their distribution in metal and age is different from those of the sample studied by McNamara (1997), we can expect that the LSQ solutions in Table 3 provide a precise description of their main properties. At present the best available observations are the MACHO data for the Galactic Bulge (Minniti et al. 1997). For these stars neither $\log T_{\rm eff}$ and [Fe/H] nor individual interstellar reddenings (for estimate of the absorption) are available. Perhaps such data can be analysed following the methods used by Sasselov et al. (1997) for Cepheids. A more attractive procedure for high-precision work is application of Strömgren $uvby\beta$ photometry. This photometric system is designed to give accurate determination of individual reddening, and as shown e.g. by McNamara it also allows precise determination of $\log T_{\rm eff}$ and [Fe/H]. Since the scatter of our new fitting formulae is around ± 0.025 mag, this method is expected to provide very precise distance moduli for sufficiently precise photometry. For δ Sct variables, $M_{\rm bol}$ is in the interval 1–2.5. With moduli to the Galactic Bulge $\simeq$ 15.0 and to LMC $\simeq$ 18.5 and absorption up to $A_V \simeq$ 0.5 high-precision photometry down to $V \simeq$ 21.5 is required. This is feasible today using medium-sized telescopes (1.5–2.5 m).

We reiterate that all applications of δ Sct stars require reliable mode identification. For HADS in LMC with metal content not very different from the galactic value we can expect that the fundamental mode is almost always excited. Comparisons of distances derived for many HADS in LMC can be used as a check. An exciting but perhaps unlikely possibility is the discovery of an LMC cluster containing both classical Cepheids, δ Sct stars and an eclipsing and spectroscopic binary which all allow precise distance determination, providing direct comparison of these three distance scales.

5.2. Population II

McNamara (1997) used a relation almost identical to Eq. (4) to determine distances to several globular clusters. However, at present we do not know to which precision Eq. (4) can be applied to the very metal-weak clusters with [Fe/H] < -2. None is available in the Hipparcos sample and only two are present in McNamara's list. It also seems premature to continue the simple theoretical modelling of the ($\log \Pi_0$, $M_{\rm bol}$) plane because we do not have safe information on the precise location of the HADS instability strip for this group. However, it is promising that McNamara obtained results that compare favourably with other data.

5.3. Main conclusions

Our main conclusions are:

(i) The average PL relation Eq. (4) can be used for distance determination of HADS with an uncertainty of ± 0.10 mag. Eq. (4) is supported both by Hipparcos data, by McNamara's

semiempirical analysis and by direct self-consistent stellar-evolution and pulsation modelling.
(ii) The accuracy can be significantly improved by introduction of colour and metallicity terms. PLCZR give an uncertainty of ± 0.025 mag.
(iii) HADS can provide improved distance determinations of the Galactic Bulge and the Magellanic Clouds when high-precision Strömgren photometry becomes available.
(iv) Other δ Sct stars can also be used when safe mode identification is available.
(v) In the near future δ Sct variables can probably also give precise distances to many globular clusters.

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