Astron. Astrophys. 352, L116–L120 (1999)

ASTRONOMY
AND
ASTROPHYSICS

Letter to the Editor

Keplerian frequencies and innermost stable circular orbits of rapidly rotating strange stars

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Received 9 September 1999 / Accepted 2 November 1999

Abstract. It has been suggested that the frequency in the co-rotating innermost stable circular orbit (ISCO) about a compact stellar remnant can be determined through X-ray observations of low-mass X-ray binaries, and that its value can be used to constrain the equation of state of ultradense matter. Upon constructing numerical models of rapidly rotating strange (quark) stars in general relativity, we find that for stars rotating at the equatorial mass-shedding limit, the ISCO is indeed above the stellar surface, for a wide range of central energy densities at a height equal to 11% of the circumferential stellar radius, which scales inversely with the square root of the energy density, $\rho_0 c^2$, of self-bound quark matter at zero presure. For these models, the ISCO frequency is $81.5 \pm 1.5\%$ of the stellar rotational frequency, whose maximum value $\Omega_K = \sqrt{3.234\,G\rho_0}$ is attained for a model close to the maximum-mass model, with $M = 2.86 M_\odot (\rho_0/4.2\times 10^{14}\,\mathrm{g\,cm^{-3}})^{-1/2}$. In contrast to static models, ISCO frequencies below 1.1 kHz are allowed – in fact, at the canonical value $\rho_0 = 4.2\times 10^{14}\,\mathrm{g\,cm^{-3}}$, the ISCO frequencies of rapidly rotating strange stars can be as low as 0.9 kHz for a $1.3 M_\odot$ strange star. Hence, the presence of strange stars in low-mass X-ray binaries cannot be excluded on the basis of the currently observed frequencies of kHz QPOs, such as the cut-off frequency of 1066 Hz in 4U 1820-30.

Key words: dense matter – stars: neutron – equation of state – X-rays: stars

1. Introduction

The discovery of millisecond variability in the flux of low-mass X-ray binaries (LMXBs) has raised the prospect of constraining the properties of matter at supranuclear densities, which is thought to make up the compact stellar remnant in these sources. Conventionally, the compact objects is taken to be a neutron star, and it has been shown (Kaaret et al., 1997; Kluźniak, 1998) how the assumption that the highest observed frequency in the X-ray flux is the orbital frequency in the innermost (marginally) stable orbit about the star (Kluźniak and Wagoner, 1985; Syunyaev and Shakura, 1986; Kluźniak et al., 1990) leads to significant constraints on the equation of state of matter at such densities, as very few models of ultra-dense matter admit neutron stars of mass high enough to allow maximum orbital frequencies as low as the observed values in the quasi-periodic oscillations (QPOs) – for instance, the QPO frequency in 4U 1820-30 saturates at 1.07 kHz (Zhang et al., 1998).

Still, the evolutionary status of LMXBs and the nature of the accreting compact object are not clear. It is known that the X-ray bursters cannot be black holes, because their photospheric radius and the temperature during the burst both tend to a definite value, thus showing the presence of a stellar surface, which is also required to explain the (type I) X-ray bursts as thermonuclear explosions of accreted material. The inferred radii (and masses) are consistent with models of neutron stars, but it is possible that the compact object is a "strange", i.e. quark, star (Cheng and Dai, 1996). If it were, at least in the sources 4U 1820-30 and 4U 1636-53, then the energy density of self-bound quark matter at zero pressure would have to have the unusually low value $\rho_0 < 4.2 \times 10^{14}\,\mathrm{g\,cm^{-3}}$, if the maximum observed frequencies of the kHz QPOs were the orbital frequencies in the ISCO about *slowly* rotating strange stars; and the observed QPO frequencies could not be the orbital frequencies at the *surface* of such stars, for any value of ρ_0 (Bulik et al., 1999).

However, it seems likely that in these very old LMXBs, the compact star has been spun up through accretion to very high[1] frequencies – in the relativistic regime, a neutron star would have to accrete only $\sim 0.2 M_\odot$ to attain angular momentum $J = 0.6 GM^2/c$ (Lipunov and Postnov, 1984; Kluźniak and Wagoner 1985). A strong magnetic field could prevent rapid rotation, but strange stars may not support such a field (Hor-

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[1] The frequency of ~ 300 Hz observed during some X-ray bursts is unlikely to be the rotational frequency of the star, as the long duration of the oscillation (~ 10 s), its harmonic content, and its variation during the burst, seem incompatible with the flux modulation being caused by a "hot spot" on the stellar surface.

vath 1999). Gravitational instabilities could also, in principle, limit the rotation rate, but the stars considered here are likely to be sufficiently cold for the r-mode instability to be inoperative (Andersson et al., 1999). We show that if LMXBs harbour rapidly rotating strange stars, the constraints from kHz QPOs on the stellar mass and on ρ_0 are relaxed to a surprising degree. We also discuss the rotational frequency of "Keplerian" models of strange stars, i.e., ones in which the rotation rate at the stellar equator is equal to the orbital frequency for the star, at the same equatorial radius.

2. Strange stars

Quark stars are likely to exist if the ground state of matter at large atomic number is in the form of a quark fluid, which would then necessarily be composed of about equal numbers of up, down, and strange quarks (Bodmer, 1971). Today, such matter is called strange matter. Its thermodynamic properties have been discussed in detail within the bag model by Farhi and Jaffe (1984) in the context of quantum chromodynamics. The first relativistic model of stars composed of quark matter was computed by (Brecher and Caporaso, 1976). The cosmological consequences of the presumed existence of strange matter were first discussed in detail by (Witten, 1984), who also showed that the maximum mass of a static strange star scales as $\rho_0^{-1/2}$ and is $2M_\odot$ for $\rho_0 = 4 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. Detailed models of strange stars have been constructed by Alcock et al. (1986) and Haensel et al. (1986).

Astrophysical implications of these ideas are not yet clear. It has been pointed out that young, glitching radio pulsars are probably neutron stars (Alpar, 1987), and that strange stars are unlikely to be present in Hulse-Taylor type binaries, as their coalescence may lead to dispersal of nuggets of quark matter which would have precluded the formation of young neutron stars in the Galaxy (Madsen 1988, Caldwell and Friedman 1991). However, there seems to be no objection to millisecond pulsars or the compact objects in LMXBs being strange stars (Kluźniak, 1994; Cheng and Dai, 1996), and it has even been suggested (Madsen 1999) that millisecond pulsars can be formed directly in supernovae, if they are strange stars [unlike neutron stars, whose rotation rate would be quickly damped by the r-mode instability: Lindblom et al. (1998), Andersson et al. (1999), Kokkotas and Stergioulas (1999)].

Our work is informed by the question whether the presence of strange stars in LMXBs can be excluded on the basis of the observed timing properties of these sources.[2] Specifically, it has been asked whether the observed frequency of the kHz QPOs in sources such as 4U 1820 −30 (1.07 kHz) is not too low to be compatible with the maximum mass of strange stars (Bulik et al., 1999). For this reason, in constructing our models of strange stars, we have focussed on an equation of state which yields the largest masses of static strange star models:

$$P = (\rho - \rho_0)c^2/3\ . \qquad (1)$$

We have also investigated the more general case, where the factor of 1/3 in Eq. (1) is replaced by a different positive constant, $a \leq 1$.

3. Keplerian models of strange stars

We have computed exact numerical models of strange stars in general relativity using the Stergioulas and Friedman (1995) code (see Stergioulas 1998 for a description). In this code, the equilibrium models are obtained following the KEH (Komatsu et al. 1989) method, in which the field equations are converted to integral equations using appropriate Green's functions.

The detailed expected properties of strange stars depend on the adopted theory of interactions. All models presented here were constructed using Eq. (1) for the equation of state. For our models of rotating stars, we find that mass and radial quantities (e.g., the stellar radius and the height of the ISCO above it) accurately scale as $\rho_0^{-1/2}$, just as for the static stars, while the frequencies scale as $\rho_0^{1/2}$. For the more general e.o.s. $P = a(\rho - \rho_0)$, we also confirm the approximate scalings with a, discovered by Lattimer et al. (1990) – to within 9% we find that for our Keplerian models, the maximum stellar mass scales as $a^{1/2}$, the stelar radius scales as $a^{1/4}$, and the rotation rate of the star scales as $a^{-1/8}$. Thus, between the scalings with ρ_0 and with a, the numerical results presented in Table 1 can at once be extended to the general e.o.s. of strange matter.

In Figs. 1 through 3, we present the mass, radius and the ISCO frequencies in our Keplerian models, for three values of ρ_0, and compare them with the static models. In Fig. 4 we present the ISCO angular frequencies as a function of the central energy density of the strange star, and also exhibit the (larger) angular frequency of the star itself. The maximal rotation rate of a strange star is very close to the rotation rate of the maximum-mass model, i.e., $9522\,\mathrm{s^{-1}}$ for $\rho_0 = 4.2 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. Note, that because of the scalings with energy density it is described by the simple formula $\Omega_K = \sqrt{3.234\,G\rho_0}$, where G is the gravitational constant. The dimensional form of this formula was anticipated by Prakash et al. (1990) and Glendenning (1990).

Table 1 presents in detail, the various stellar parameters obtained in our calculation for Keplerian strange stars, modeled with Eq. (1) for the value $\rho_0 = 4.2 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. In addition to the central density, gravitational mass of the star, its radius and maximal rotation rate, the successive columns list the angular frequency ($\Omega_+ \equiv 2\pi f_+$) in the co-rotating ISCO (at height h_+ above the surface), the height of the retrograde ISCO, the stellar angular momentum, the moment of inertia, the ratio of the polar to equatorial radii, and the polar and the equatorial (forward and backward) redshifts. The ratio of kinetic to potential energy T/W is much larger for these models than for neutron stars. Note that in all cases, the co-rotating ISCO is above the stellar surface and, as exhibited in Fig. 3, the ISCO frequencies are much lower for the Keplerian models than for

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[2] In the conversion of accreted matter to quark matter, substantial energy will be released at the base of the crust. However, since the mass accretion rate in LMXBs is only inferred from the X-ray luminosity, there is no direct way of differentiating this from the gravitational binding energy release, if the conversion of matter is proceeding at a quasi-steady rate.

Table 1. Keplerian models of strange stars, for $\rho_0 = 4.2 \times 10^{14}\mathrm{g\,cm^{-3}}$.

ρ_c $10^{14}\mathrm{g\,cm^{-3}}$	M $M_\odot$	R km	Ω_K $10^3\,\mathrm{s^{-1}}$	Ω_+ $10^3\,\mathrm{s^{-1}}$	h_+ km	h_- km	J $GM_\odot^2/c$	I $10^{45}\mathrm{g\,cm^2}$	r_p/r_e	T/W	z_p	z_b	$-z_f$
5.000	1.336	16.81	7.049	5.664	1.86	9.32	2.349	2.929	0.3192	0.2785	0.2731	0.8931	0.2821
5.250	1.598	17.29	7.257	5.877	1.90	11.2	3.123	3.782	0.3365	0.2709	0.3279	1.045	0.2942
5.500	1.800	17.62	7.432	6.051	1.87	12.7	3.760	4.447	0.3500	0.2645	0.3739	1.176	0.3048
5.750	1.963	17.75	7.591	6.217	1.86	13.9	4.299	4.978	0.3630	0.2590	0.4145	1.294	0.3113
6.105	2.161	17.89	7.786	6.389	1.88	15.6	5.003	5.647	0.3750	0.2534	0.4676	1.453	0.3199
6.500	2.304	17.87	7.979	6.582	1.84	16.7	5.474	6.030	0.3906	0.2465	0.5121	1.589	0.3260
6.600	2.336	17.87	8.012	6.615	1.84	16.9	5.590	6.131	0.3930	0.2457	0.5221	1.619	0.3270
6.750	2.388	17.86	8.080	6.678	1.84	17.4	5.788	6.296	0.3969	0.2441	0.5393	1.674	0.3291
7.275	2.520	17.79	8.280	6.864	1.82	18.6	6.258	6.643	0.4094	0.2387	0.5875	1.829	0.3348
8.668	2.730	17.51	8.708	7.229	1.79	20.5	7.007	7.072	0.4312	0.2285	0.6835	2.153	0.3458
9.023	2.753	17.40	8.803	7.327	1.76	20.7	7.055	7.043	0.4367	0.2257	0.6999	2.207	0.3474
9.658	2.796	17.23	8.958	7.459	1.75	21.1	7.180	7.045	0.4437	0.2221	0.7290	2.309	0.3501
10.33	2.823	17.06	9.103	7.582	1.74	21.3	7.233	6.983	0.4504	0.2186	0.7535	2.394	0.3523
11.06	2.844	16.89	9.245	7.699	1.73	21.5	7.259	6.901	0.4562	0.2154	0.7760	2.474	0.3544
11.84	2.854	16.69	9.385	7.823	1.72	21.6	7.232	6.772	0.4625	0.2119	0.7953	2.541	0.3555
12.67	2.861	16.52	9.522	7.933	1.71	21.7	7.211	6.656	0.4672	0.2090	0.8140	2.608	0.3573
13.56	2.860	16.33	9.648	8.047	1.69	21.7	7.124	6.489	0.4727	0.2056	0.8271	2.652	0.3579
14.51	2.852	16.14	9.782	8.153	1.69	21.7	7.038	6.323	0.4773	0.2026	0.8415	2.702	0.3589
15.53	2.842	15.97	9.909	8.256	1.68	21.6	6.937	6.153	0.4812	0.1998	0.8532	2.741	0.3600
16.63	2.829	15.78	10.03	8.363	1.67	21.5	6.829	5.980	0.4852	0.1970	0.8638	2.777	0.3606

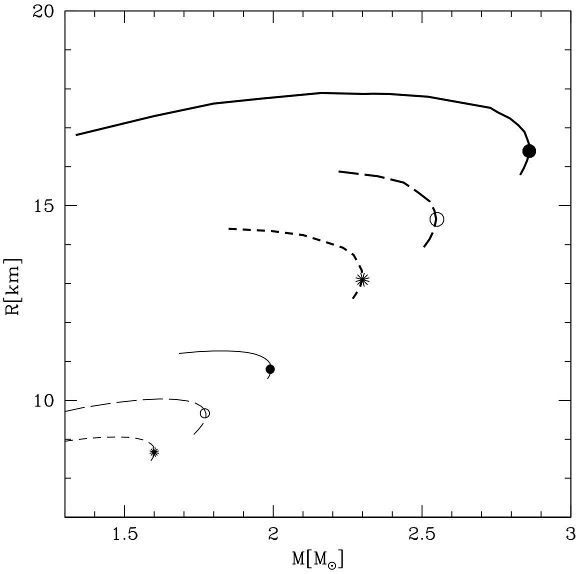

Fig. 1. Radius vs. mass for strange stars (near their maximum mass). Models of both non-rotating stars (thin lines) and maximally rotating stars (thick lines, i.e., upper three curves) are shown for three values of the energy density of quark matter at zero pressure, ρ_0: $4.2\times10^{14}\,\mathrm{g\,cm^{-3}}$ (continuous lines), $5.3\times10^{14}\,\mathrm{g\,cm^{-3}}$ (long-dashed lines), $6.5\times10^{14}\,\mathrm{g\,cm^{-3}}$ (dashed lines). The maximum-mass models for these values of ρ_0 are indicated by a filled circle, an open circle and a star, respectively. Note that the radius and mass scale as $\rho_0^{-1/2}$. A large increase of the radius and maximum mass is evident as the stellar rotation rate increases from zero to the equatorial mass-shedding limit.

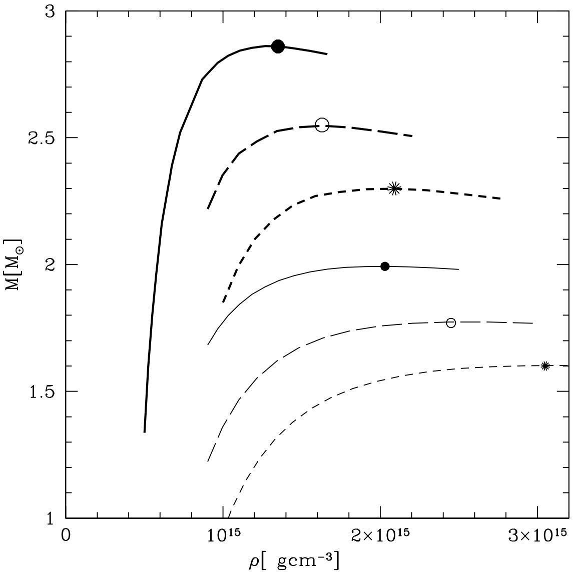

Fig. 2. Mass vs. central energy density for the same models as in Fig. 1. For each sequence, a star is stable only at values of ρ smaller than, approximately, the one corresponding to the maximum mass.

static models (at fixed stellar mass) and differ considerably from their lowest-order slow-rotation approximation. The significant departure from the slow-rotation result is explained by the unusually large oblateness of rapidly rotating strange stars, and the fact that the ISCO frequency and height depend not only on the angular momentum, but also on the stationary quadrupole moment, in rapidly rotating stars (Shibata and Sasaki, 1998; Sibgatullin and Sunayev, 1998).

4. Conclusions

We have calculated exact models of rotating strange stars – these computations are in excellent agreement (Table 2) with

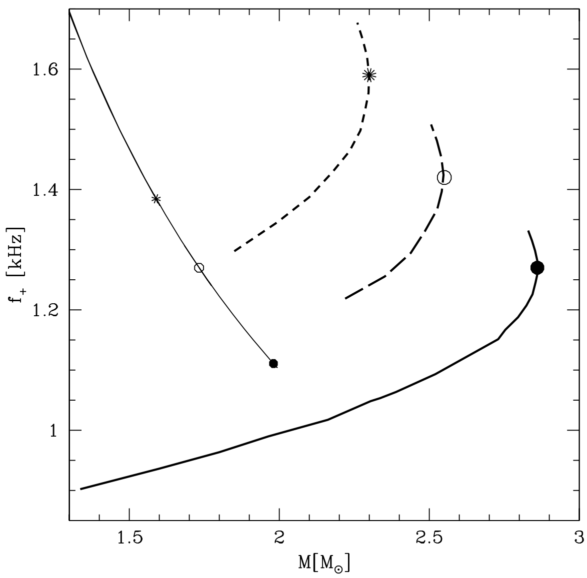

Fig. 3. The frequency of the co-rotating innermost stable circular orbit as a function of mass for static models (thin, continuous line) and for strange stars rotating at the equatorial mass-shedding limit (thick lines, in the style of Fig. 1). For the static models, this frequency is given by the keplerian value at $r = 6GM/c^2$, i.e., by $f_+ = 2198\,\mathrm{Hz}(M_\odot/M)$, and the minimum ISCO frequency corresponds to the maximum mass, denoted by a filled circle, an empty circle, and a star, respectively for $\rho_0/(10^{14} g\, cm^{-3}) =$ 4.2, 5.3, and 6.5. Note that the ISCO frequencies for rapidly rotating strange stars can have much lower values, and $f_+ < 1$ kHz can be achieved for strange stars of fairly modest mass, e.g. $1.4 M_\odot$, if the star rotates close to the equatorial mass-shedding limit.

Table 2. Comparison of two codes.

		This work	Gourgoulhon et al.	diff. [%]
ρ_c	10^{15}g cm^{-3}	1.261	1.261	
M	$M_\odot$	2.83392	2.831	0.1
R	km	16.4252	16.54	0.7
Ω	10^3 s^{-1}	9.56261	9.547	0.2
Ω_K	10^3 s^{-1}	9.57637	9.547	0.3
T/W		0.210091	0.210	0.04
$cJ/GM_\odot^2$		7.09331	7.084	0.1
I	10^{45}g cm^2	6.51906	6.534	0.2
z_p		0.807995	0.8070	0.1
z_b		2.58820	2.584	0.2
$-z_f$		0.356272	0.3608	1.2
r_p/r_e		0.466000	0.4618	0.9

the very recent results obtained by a highly accurate code based on spectral methods (Gourgoulhon et al., 1999). We found the scalings of M, R, Ω with the parameters a, and ρ_0 in the equation of state of self-bound quark matter $P = a(\rho - \rho_0)$. In addition, we calculate the innermost stable orbits and find that, unlike for static models, for strange stars rotating at the equatorial mass-shedding limit the orbital frequencies can extend to values below 1.1 kHz. For the same models the radius of the ISCO is about

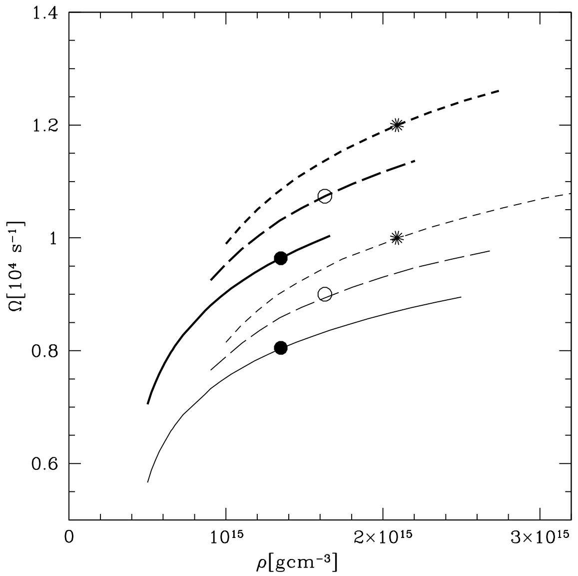

Fig. 4. The angular frequency of strange stars (the upper three, thick, lines) rotating at the equatorial mass-shedding limit, and of their co-rotating ISCOs (thin lines), as a function of central energy density. The symbols have the same meaning as in Fig. 1.

11% larger than the circumferential stellar radius, independently of the central density, or the value of ρ_0.

Our results show that the highest observed QPO frequencies in low-mass X-ray binaries (such as 1.07 kHz in 4U 1820-30) could be the orbital frequencies in the innermost stable circular orbit about strange stars, if only the stars are rotating sufficiently rapidly (as is expected in these old accreting sources). Thus, the compact objects in LMXBs could, in principle, be rapidly rotating strange stars. Further conclusions about the nature of LMXBs and of the kHz QPOs would be possible, if either the mass or the rotational period of the accreting stellar remnant were known.

Acknowledgements. This research was supported in part by the NSF grant PHY94-07194.

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